# MOVING CHARGES AND MAGNETISM FORMULAE

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## Magnetic Induction Produced By A Current (Biot-Savart Law):

The magnetic induction dB produced by an element $dl$ carrying a current I at a distance r is given by:

$$dB = \frac{\mu_0 \mu_r}{4\pi} \frac{I\, d\ell \sin\theta}{r^2} \Rightarrow \vec{dB} = \frac{\mu_0\, \mu_r}{4\pi} \frac{I\left(d\vec{\ell} \times \vec{r}\right)}{r^3}$$

here the quantity $Id\ell$ is called as current element strength.

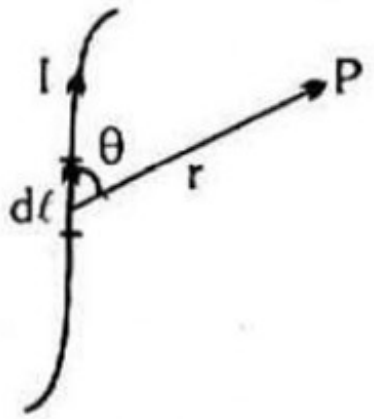
$\mu$ = permeability of the medium = $\mu_0 \mu_r$

$\mu_0$ = permeability of free space .

$\mu_r$ = relative permeability of the medium (Dimensionless quantity).

Unit of $\mu_0$ & $\mu$ is $NA^{-2}$ or $Hm^{-1}$ ;

$\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$

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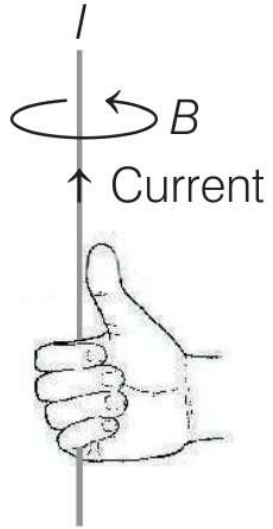
The direction of magnetic field lines can be obtained by Right Hand Thumb Rule.

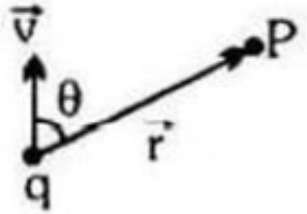
## Right Hand Thumb Rule

If we hold a current carrying conductor in the grip of the right hand in such a way that thumb points in the direction of current, then curling of fingers represents the direction of magnetic field lines.



## MAGNETIC INDUCTION DUE TO A MOVING CHARGE

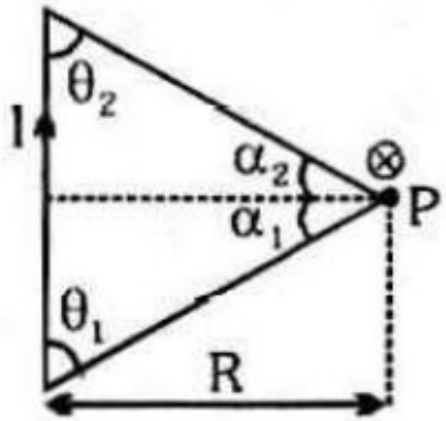
$$dB_P = \frac{\mu_0 q v \sin\theta}{4\pi r^2}$$




In vector form it can be written as $\vec{dB} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$

## MAGNETIC INDUCTION DUE TO A CURRENT CARRYING STRAIGHT CONDUCTOR

- **Magnetic induction due to a current carrying straight wire**




$$B = \frac{\mu_0 I}{4\pi R}(\cos\theta_1 + \cos\theta_2) = \frac{\mu_0 I}{4\pi R}(\sin\alpha_1 + \sin\alpha_2)$$

If the wire is very long $\theta_1 \cong \theta_2 \cong 0°$ then , $B = \frac{\mu_0 I}{2\pi R}$

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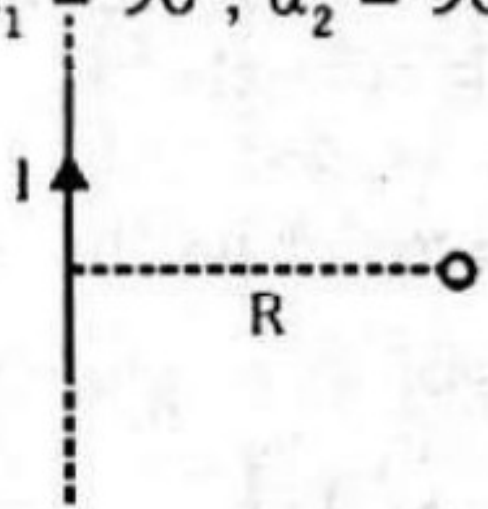
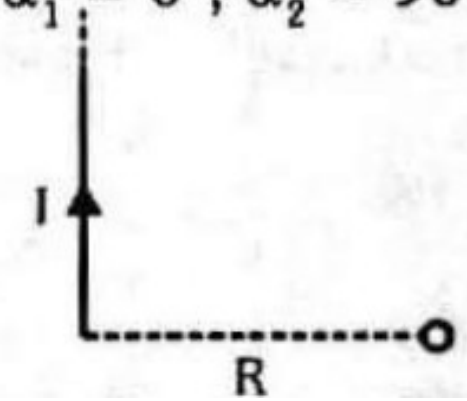
**Magnetic induction due to a infinitely long wire** $B = \frac{\mu_0 I}{2\pi R} \otimes$

$\alpha_1 = 90°; \alpha_2 = 90°$



**MAGNETIC INDUCTION DUE TO SEMI INFINITE STRAIGHT CONDUCTOR** $B = \frac{\mu_0 I}{4\pi R} \otimes$

$\alpha_1 = 0°; \alpha_2 = 90°$

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## MAGNETIC FIELD DUE TO A FLAT CIRCULAR COIL CARRYING A CURRENT :

(i) At its centre $B = \frac{\mu_0 NI}{2R}$ ⊙

where

N = total number of turns in the coil

I = current in the coil

R = Radius of the coil

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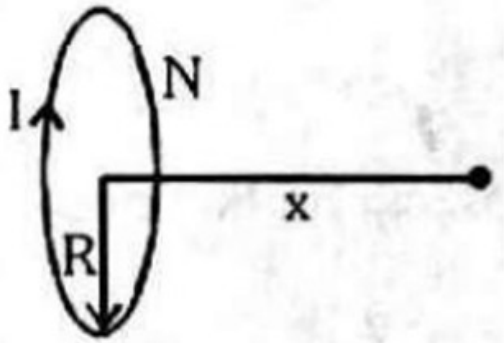
(iii) On the axis $B = \dfrac{\mu_0 NIR^2}{2\left(x^2+R^2\right)^{3/2}}$

Where x = distance of the point from the centre .

It is maximum at the centre $B_c = \dfrac{\mu_0 NI}{2R}$

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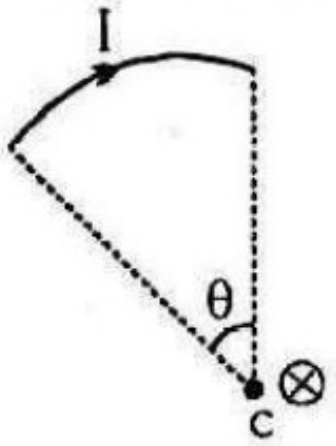
**(iii) MAGNETIC INDUCTION DUE TO FLAT CIRCULAR ARC :** $B = \frac{\mu_0 I\theta}{4\pi R}$

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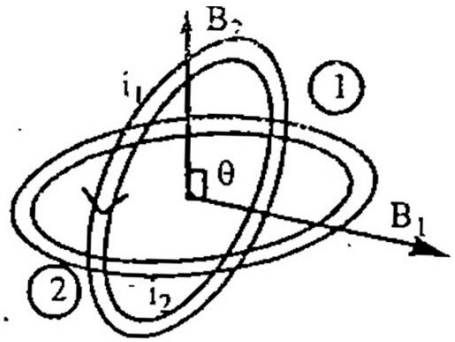
If two concentric circular loops carrying current $i_1$, $i_2$ and their planes are inclined at an angle $\theta$, the resultant magnetic induction at the common centre is




$$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2 \cos \theta}$$

where $B_1 = \frac{\mu_0 i_1}{2r_1}$, $B_2 = \frac{\mu_0 i_2}{2r_2}$

a) If $\theta = 90^0$ then $B = \sqrt{B_1^2 + B_2^2}$

b) If $\theta = 0^0$, and the coils carrying current in same direction, then $B = B_1 + B_2$

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## Magnetic field due to infinite long solid cylindrical conductor of radius R

For $r \geq R : B = \frac{\mu_0 I}{2\pi r}$

For $r < R : B = \frac{\mu_0 I r}{2\pi R^2}$

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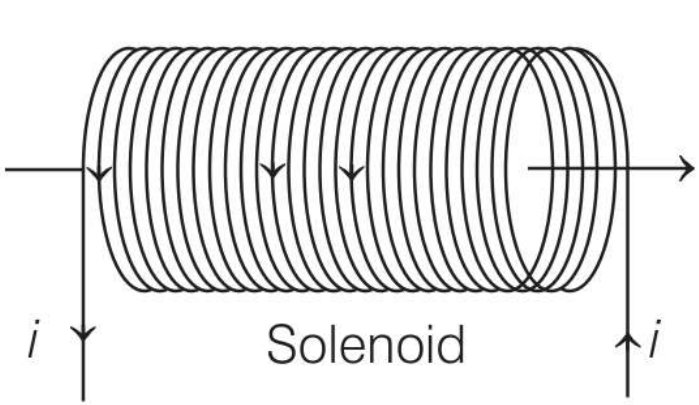
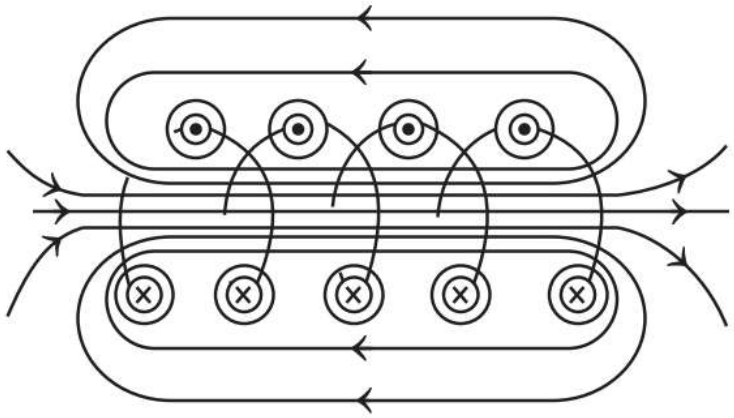
## MAGNETIC INDUCTION DUE TO SOLENOID

# Solenoid

A solenoid is a closely wound helix of insulated copper wire.

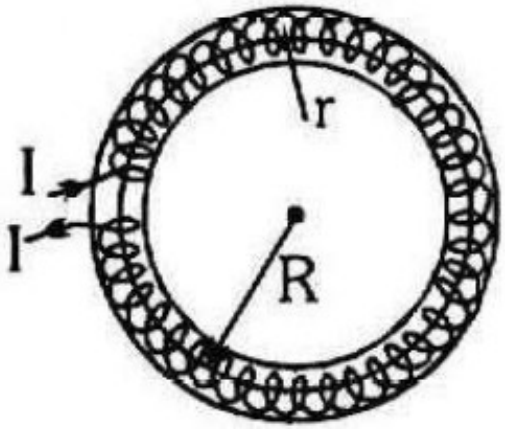


Magnetic field at a point well inside a long solenoid is given by

$$B = \mu_0 nI$$

## MAGNETIC INDUCTION DUE TO TOROID : $B = \mu_0 nI$




where $n = \frac{N}{2\pi R}$ (no. of turns per m)

N = total turns $R >> r$

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## AMPERES LAW

$$\oint \vec{B} \cdot d\vec{\ell} = \mu \Sigma I$$ where $\Sigma I$ = algebraic sum of all the currents.

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## MAGNETIC INTERACTION FORCE BETWEEN TWO PARALLEL LONG STRAIGHT CURRENTS :

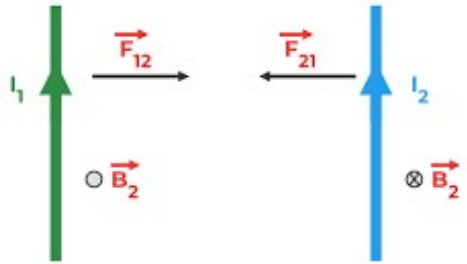
When two long straight linear conductors are parallel and carry a current in each , they magnetically interact with each other, one experiences a force. This force is of:

(i) Repulsion if the currents are anti-parallel (i.e. in opposite direction) or

(ii) Attraction if the currents are parallel (i.e. in the same direction)

This force per unit length on either conductor is given by $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$.

Where r = perpendicular distance between the parallel conductors



## MAGNETIC TORQUE ON A CLOSED CURRENT CIRCUIT :

When a plane closed current circuit of 'N' turns and of area 'A' per turn carrying a current I is placed in uniform magnetic field , it experience a zero net force, but experience a torque given by $\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{M} \times \vec{B} = BINA\sin\theta$

where $\vec{A}$ = area vector outward from the face of the circuit where the current is anticlockwise, $\vec{B}$ = magnetic induction of the uniform magnetic field.

$\vec{M}$ = magnetic moment of the current circuit = $IN\vec{A}$

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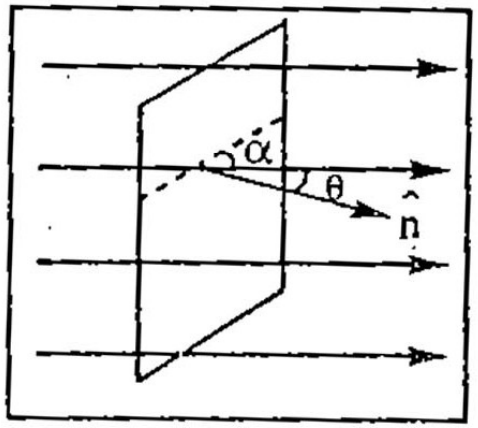
## TORQUE ACTING ON A CURRENT LOOP KEPT IN UNIFORM MAGNETIC FIELD :

When a coil carrying current is placed in uniform magnetic field, the net force on it is zero but it experiences a torque or couple.




Here $\alpha + \theta = 90^0$

Torque acting on a current carrying coil placed in uniform magnetic field is $\bar{\tau} = \bar{M} \times \bar{B}$

Torque acting on the coil is $\tau$ = Bin A $\sin\theta$

If the plane of coil is parallel to the direction of magnetic field $\tau = \tau_{max} = BinA$

If the plane of coil is perpendicular to the direction of magnetic field, $\tau = 0$

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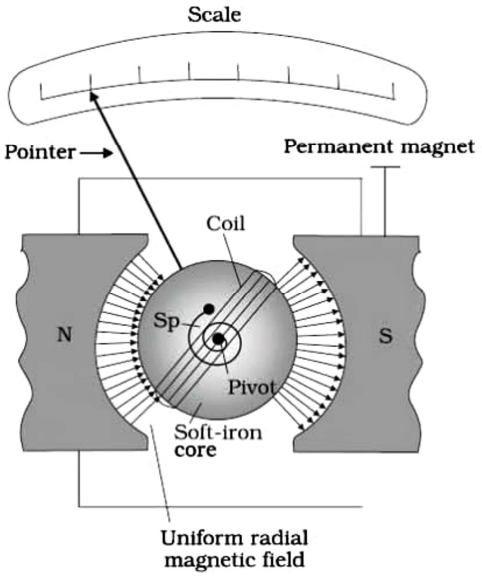
## MOVING COIL GALVANOMETER :

It consists of a plane coil of many turns suspended in a radial magnetic feild. When a current is passed in the coil it experiences a torque which produces a twist in the suspension.

This deflection is directly proportional to the torque $\therefore NIAB = K\theta$

$I = \left(\frac{K}{NAB}\right)\theta$; K = elastic torsional constant of the suspension

$I = C\theta$ $\qquad C = \frac{K}{NAB}$ = Galvanometer Constant

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Phosphor Bronze has

a) high Young's modulus so that the wire will not be stretched easily.

b) low rigidity modulus so that the wire can be twisted easily.

c) small elastic after effect so that it comes back quickly to orginal position after withdrawing current.

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Current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it. $S_I = \frac{d\theta}{di} = \frac{BAN}{C}$.

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$$S_I = \frac{d\theta}{di} = \frac{BAN}{C}$$

Sensitivity of a MCG can be increased by

i) Increasing B ii) Increasing A

iii) Increasing N iv) Decreasing C

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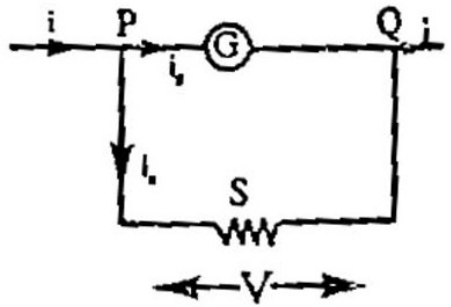
## SHUNT :

A low resistance connected in parallel to galvanometer to protect it from large current is known as shunt.

When shunt is connected

range increases but

sensitivity decreases.




$$R_{equivalent} = \frac{GS}{G+S}$$

$$V = i\,R_{eq} = i\frac{GS}{G+S}$$

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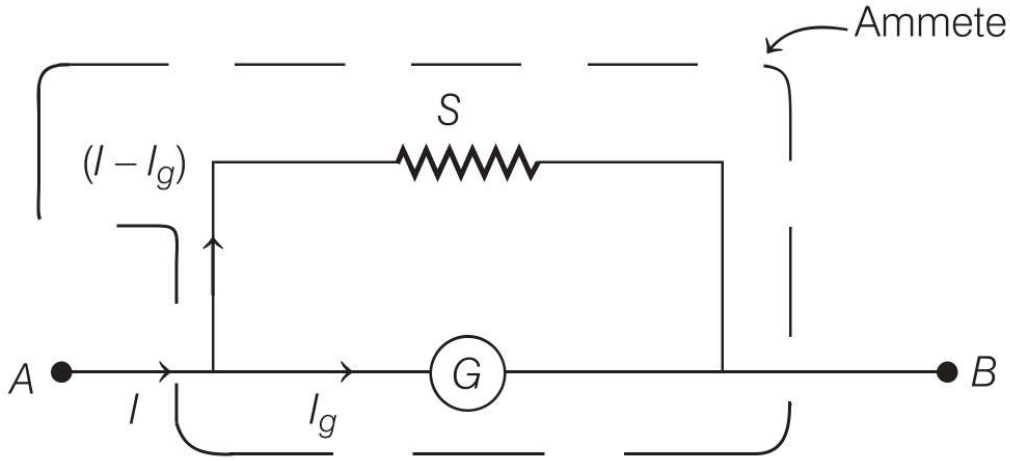
## Conversion of a Galvanometer into an Ammeter

A galvanometer can be converted into an ammeter by connecting a low resistance into its parallel. If $G$ is the resistance of a galvanometer and it give full scale deflection for current $I_g$, then required low resistance $S$, connected in its parallel for converting it into an ammeter of range $I$ is given by

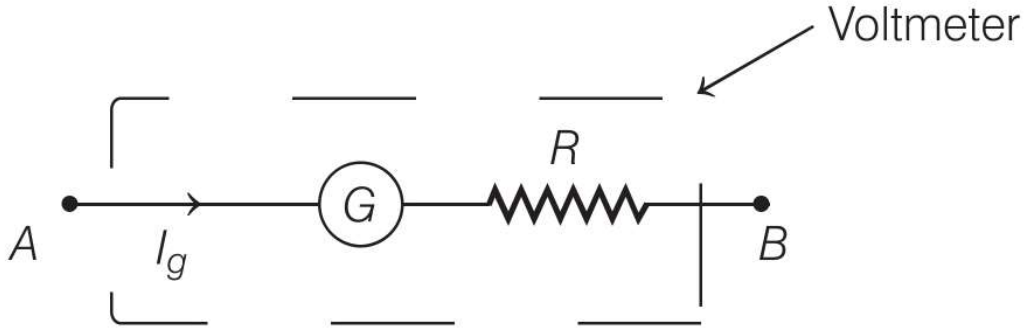



$$I_g \times G = (I - I_g) \times S \Rightarrow S = \left( \frac{I_g}{I - I_g} \right) G$$

The resistance of an ideal ammeter is zero.

## Conversion of a Galvanometer into a Voltmeter

A galvanometer can be converted into a voltmeter by connecting a high resistance into its series.



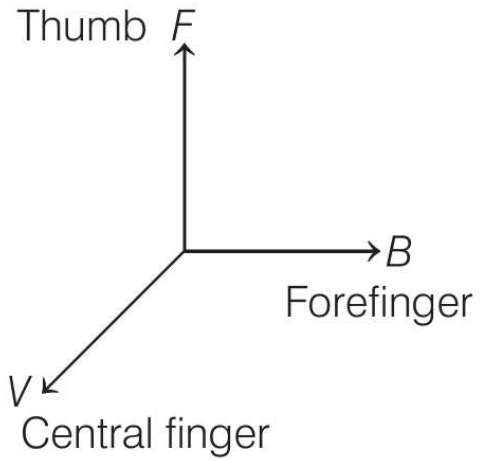

If a galvanometer of resistance $G$ shows full scale deflection for current $I_g$, then required high resistance $R$, connected in series for converting it into a voltmeter of range $V$ is given by

$$V = I_g\,(G + R)$$

$$\Rightarrow \qquad R = \frac{V}{I_g} - G$$

## Fleming's Left Hand Rule

If we stretch the thumb, the forefinger and the central finger of left hand in such a way that all three are perpendicular to each other, then if forefinger represents the direction of magnetic field, central finger represents the direction of current flowing through the conductor, then thumb will represent the direction of magnetic force.



## MOTION OF A CHARGE IN UNIFORM MAGNETIC FIELD :

(a) When $\vec{v}$ is $\parallel$ to $\vec{B}$ : Motion will be in a straight line and $\vec{F} = 0$

(b) When $\vec{v}$ is $\perp$ to $\vec{B}$ : Motion will be in circular path with radius

$$R = \frac{mv}{qB} \text{ and angular velocity } \omega = \frac{qB}{m} \text{ and } F = qvB.$$

(c) When $\vec{v}$ is at $\angle\theta$ to $\vec{B}$ : Motion will be helical with radius

$$R_k = \frac{mv\sin\theta}{qB} \text{ and pitch } P_H = \frac{2\pi mv\cos\theta}{qB} \text{ and } F = qvB\sin\theta.$$

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## LORENTZ FORCE :

An electric charge 'q' moving with a velocity $\vec{v}$ through a magnetic field of magnetic induction $\vec{B}$ experiences a force $\vec{F}$, given by $\vec{F}=q\vec{v}\times\vec{B}$. There fore, if the charge moves in a space where both electric and magnetic fields are superposed .

$\vec{F}$ = net electromagnetic force on the charge = $q\vec{E}+q\vec{v}\times\vec{B}$

This force is called the **LORENTZ FORCE**

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## MOTION OF CHARGE IN COMBINED ELECTRIC FIELD & MAGNETIC FIELD

- When $\vec{v} \| \vec{B}$ & $\vec{v} \| \vec{E}$, motion will be uniformly accelerated in straight line as $F_{magnetic} = 0$ and $F_{electrostatic} = qE$
  So the particle will be either speeding up or speeding down
- When $\vec{v} \| \vec{B}$ & $\vec{v} \perp \vec{E}$, motion will be uniformly accelerated in a parabolic path
- When $\vec{v} \perp \vec{B}$ & $\vec{v} \perp \vec{E}$, the particle may more undeflected & undeivated with same uniform speed if $v = \frac{E}{B}$ (This is called as velocity selector condition)

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**MAGNETIC FORCE ON A STRAIGHT CURRENT CARRYING WIRE :** $\vec{F} = I(\vec{L} \times \vec{B})$

$I$ = current in the straight conductor

$\vec{L}$ = length of the conductor in the direction of the current in it

$\vec{B}$ = magnetic induction. (Uniform throughout the length of conductor)

**Note :** In general force is $\vec{F} = \int I(d\vec{\ell} \times \vec{B})$

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# Motion of a Charged Particle in a Uniform Magnetic Field

When the charged particle enters parallel or anti-parallel to the magnetic field, then its follows a straight line path.

When charged particle enters normally to the magnetic field it follows a circular path.

The radius of the path, $r = \frac{mv}{Bq}$

$$r \propto mv \text{ and } r \propto \frac{1}{(q/m)}$$

Time period, $T = \frac{2\pi m}{Bq}$

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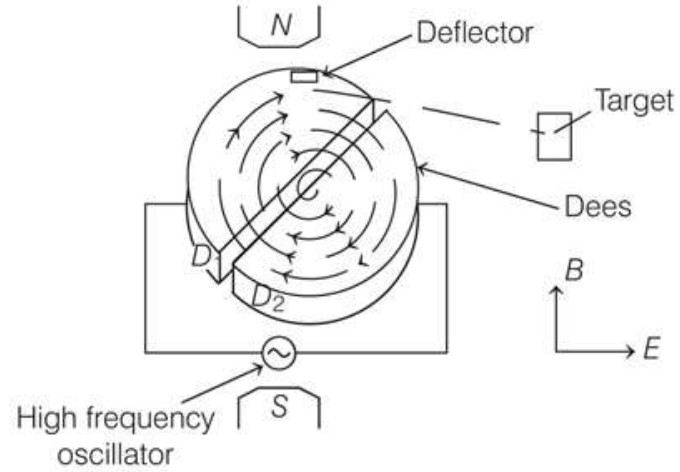
## Principle of Cyclotron

A positively charged particle can be accelerated through a moderate electric field by crossing it again and again by use of strong magnetic field.




Radius of circular path, $r = \dfrac{mv}{Bq}$

Cyclotron frequency, $\nu = \dfrac{Bq}{2\pi m}$

where, $m$ and $q$ are mass and charge of the positive ion and $B$ is strength of the magnetic field.

Maximum kinetic energy gained by the particle, $E_{max} = \dfrac{B^2 q^2 r_0^2}{2m}$

where, $r_0$ = maximum radius of circular path.

If a charge 'q' is moving in a circular orbit of radius 'r' with uniform speed 'v' making 'f' rotations per second, with time period 'T' and angular velocity ω

a) current $i = fq = \frac{q}{T} = \frac{vq}{2\pi r} = \frac{\omega q}{2\pi}$

b) Equivalent magnetic moment

$$M = i\left(\pi r^2\right) = fq\left(\pi r^2\right) = \frac{\omega q r^2}{2} = \frac{\pi r^2 q}{T} = \frac{vqr}{2}$$

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## Magnetic dipole

- Magnetic moment $M = m \times 2\ell$ where m = pole strength of the magnet
- Magnetic field at axial point (or End-on) of dipole $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{r^3}$
- Magnetic field at equatorial position (Broad-on) of dipole $\vec{B} = \frac{\mu_0}{4\pi}\frac{(-\vec{M})}{r^3}$
- At a point which is at a distance r from dipole midpoint and making angle $\theta$ with dipole axis.

Magnetic Potential $$V = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}$$

Magnetic field $$B = \frac{\mu_0}{4\pi}\frac{M\sqrt{1+3\cos^2\theta}}{r^3}$$

- Torque on dipole placed in uniform magnetic field $\vec{\tau} = \vec{M} \times \vec{B}$
- Potential energy of dipole placed in an uniform field $U = -\vec{M} \cdot \vec{B}$

## Magnetic dipole

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Magnetic Potential $$V = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}$$

Magnetic field $$B = \frac{\mu_0}{4\pi}\frac{M\sqrt{1+3\cos^2\theta}}{r^3}$$

- Torque on dipole placed in uniform magnetic field $\vec{\tau} = \vec{M} \times \vec{B}$
- Potential energy of dipole placed in an uniform field $U = -\vec{M} \cdot \vec{B}$

- **Intensity of magnetisation** $I = M/V$
- **Magnetic induction** $B = \mu H = \mu_0(H + I)$
- **Magnetic permeability** $\mu = \frac{B}{H}$
- **Magnetic susceptibility** $\chi_m = \frac{I}{H} = \mu - 1$

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- **Curie law**
  - For paramagnetic materials $\chi_m \propto \frac{1}{T}$
- **Curie Wiess law**
  - For Ferromagnetic materials $\chi_m \propto \frac{1}{T - T_C}$

  Where $T_c$ = curie temperature

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# THANK YOU

# JAI HIND

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