ELECTROMAGNETIC INDUCTION FORMULAE

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Magnetic Flux

The total number of magnetic field lines crossing through any surface normally, when it is placed in a magnetic field is known as magnetic flux of that surface.

$$d\phi = \mathbf{B} \cdot d\mathbf{s} = Bds \cos\theta$$

Its SI unit is tesla-metre (or weber)

$$\text{CGS unit of } \phi = \text{maxwell, } 1 \text{ weber} = 10^8 \text{ maxwell},$$

Dimensional formula of magnetic flux

$$[\phi] = [ML^2 T^{-2} A^{-1}]$$

Faraday's Laws of Electromagnetic Induction

Whenever the magnetic flux linked with a circuit changes, an induced emf is produced in it.

The induced emf lasts, so long as the change in magnetic flux continues.

The magnitude of induced emf is directly proportional to the rate of change in magnetic flux, *i.e.*

$$e \propto \frac{d\phi}{dt} \quad \Rightarrow \quad e = -\frac{d\phi}{dt}$$

Induced current is given as

$$I = \frac{1}{R}\left(\frac{d\phi}{dt}\right)$$

Induced charge is given as $q = \frac{1}{R}(d\phi)$.

If induced current is produced in a coil rotated in uniform magnetic field, then

$$I = \frac{NBA\,\omega \sin \omega t}{R} = I_0 \sin \omega t$$

where, $I_0 = NBA\omega$ = peak value of induced current,

N = number of turns in the coil ,

B = magnetic induction,

ω = angular velocity of rotation and

A = area of cross-section of the coil.

Lenz's Law

The direction of induced emf or induced current is always in such a way, that it opposes the cause due to which it is produced.

Lenz's law is in accordance with the conservation of energy.

Direction of the induced current can be determined as

(i) if flux is decreasing, the magnetic field due to induced current will be along the existing magnetic field.

(ii) if flux is increasing, the magnetic field due to induced current will be opposite to existing magnetic field.

Lenz's law:

The direction of the induced current is such that **it opposes the original change** producing it.

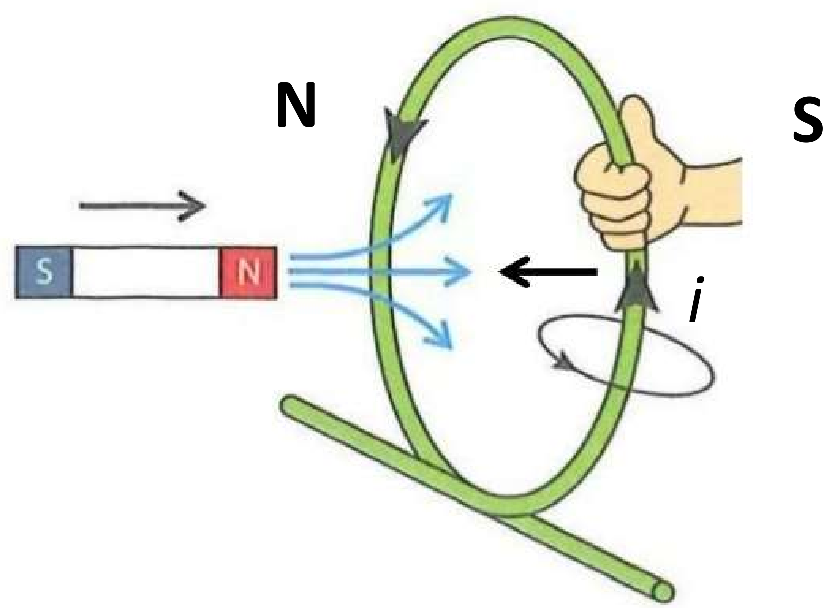


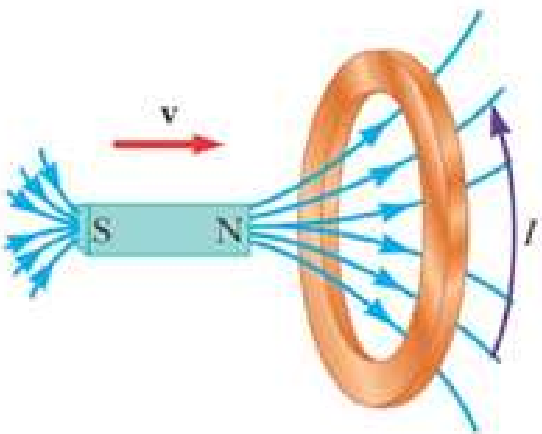
Image source: http://ffden-2.phys.uaf.edu/

$$e = \frac{-\,d\phi_B}{dt}$$

$$(\because \ rate\ of\ change\ of\ fulx = \frac{\phi_2 - \phi_1}{t_2 - t_1})$$

If the closed coil contains N number of turns, then

$$e = -N\,\frac{d\phi_B}{dt}$$



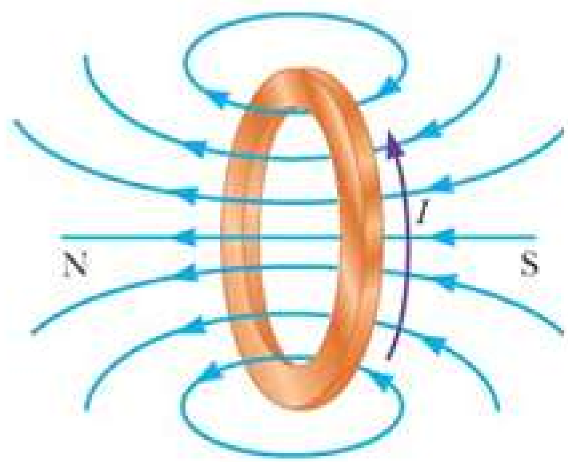
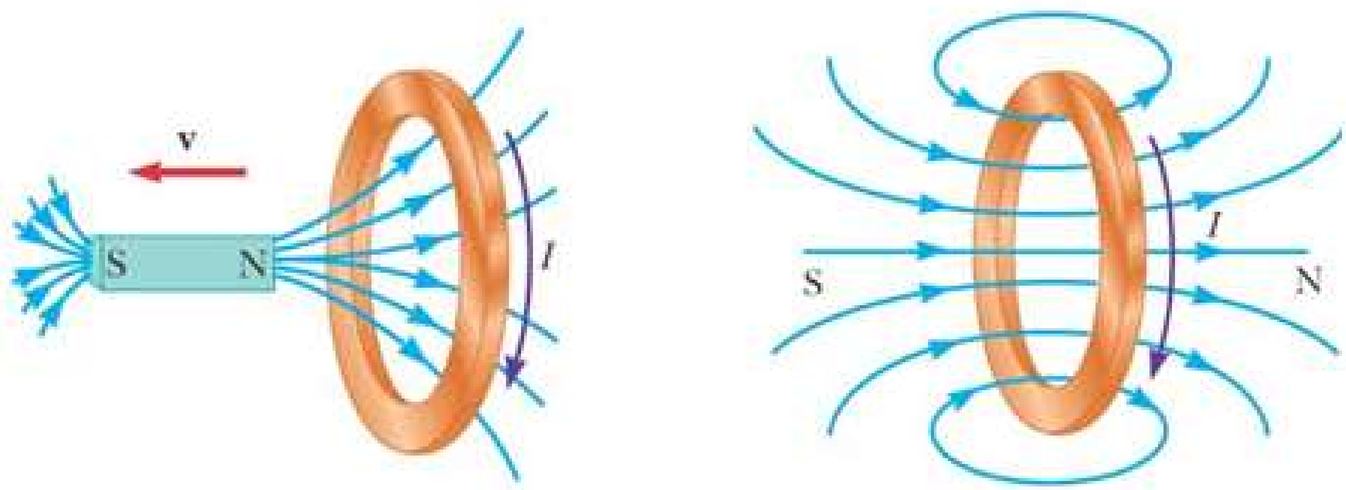


Image source: https://physicsabout.com/lenzs-law/



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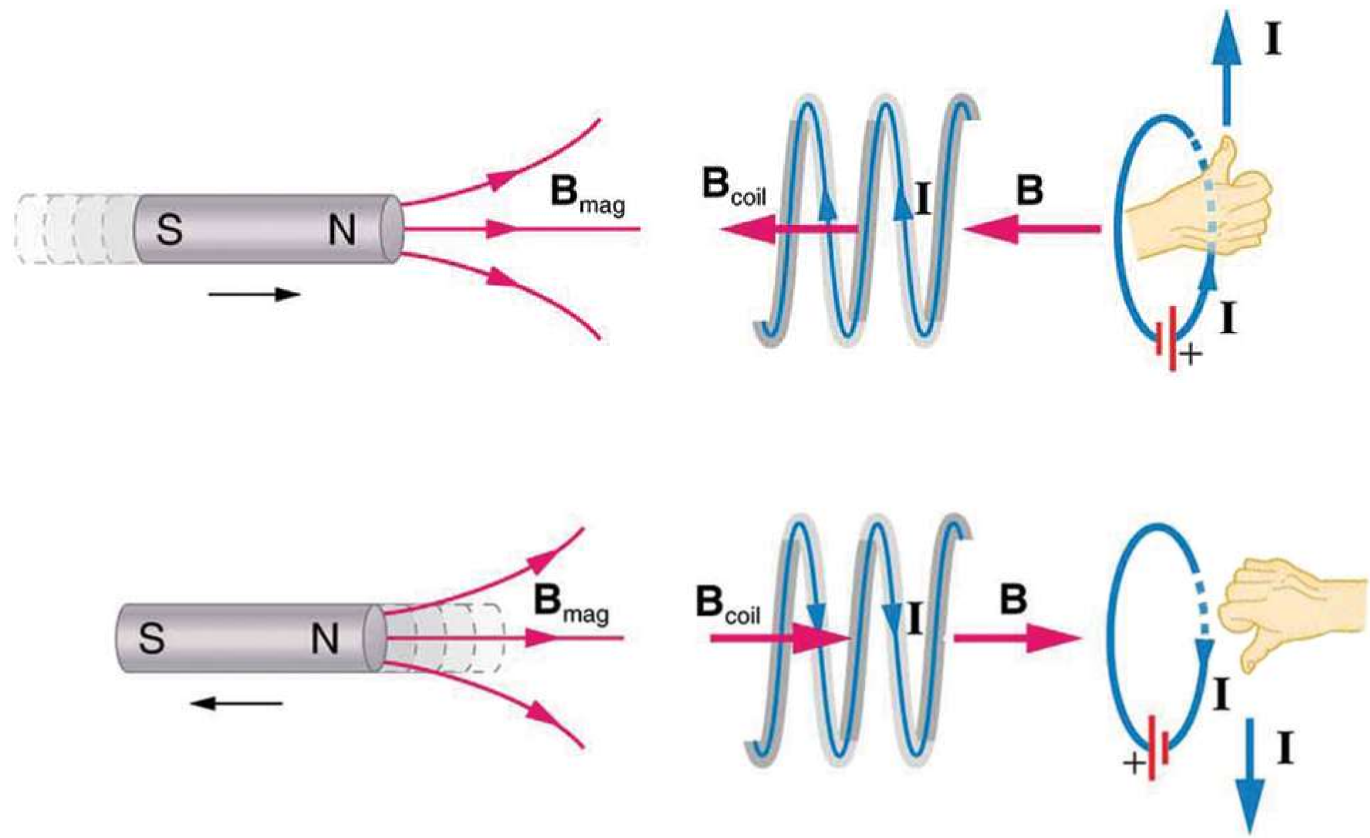

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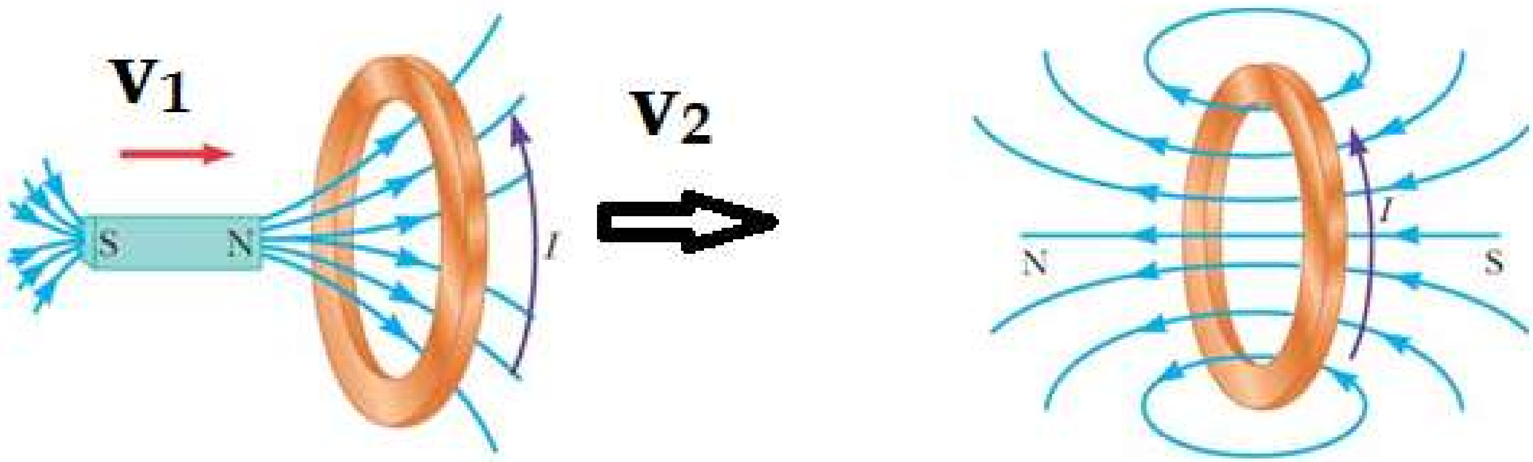
Special Cases:

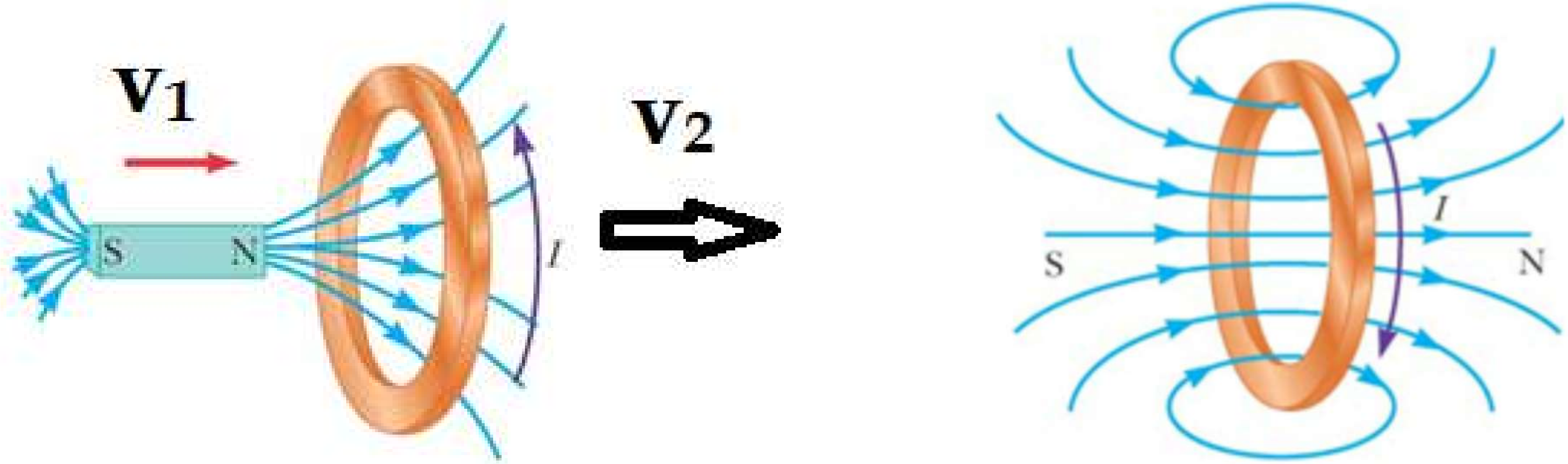
Case 1:

If $v_1 > v_2$



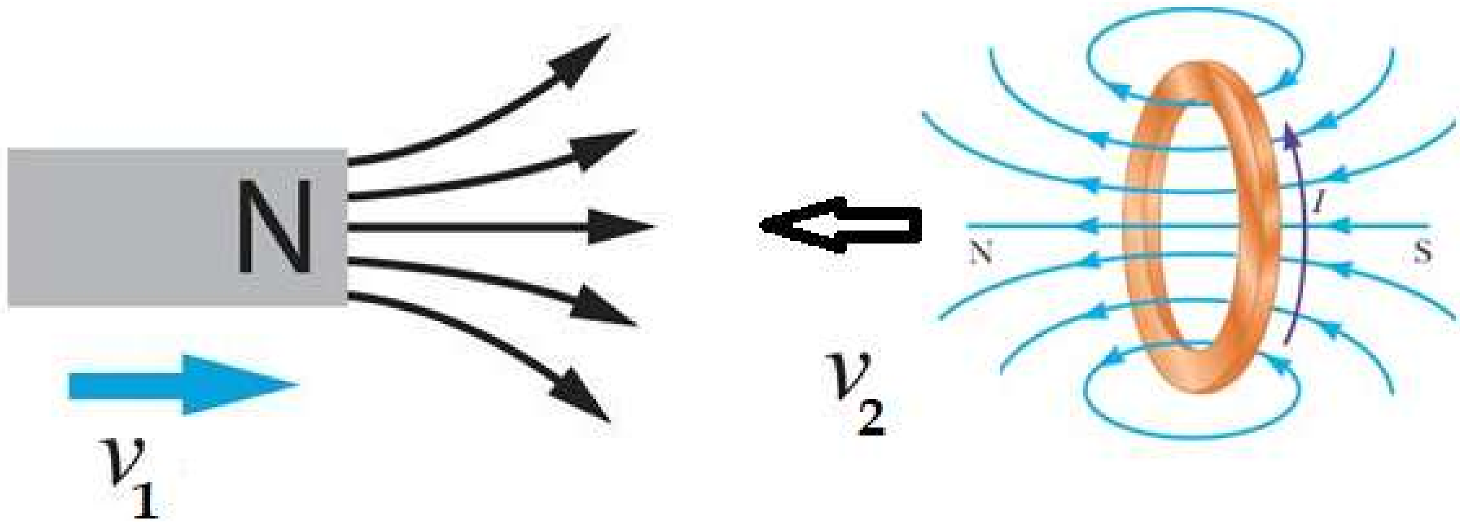
Case 2:

If $v_1 < v_2$



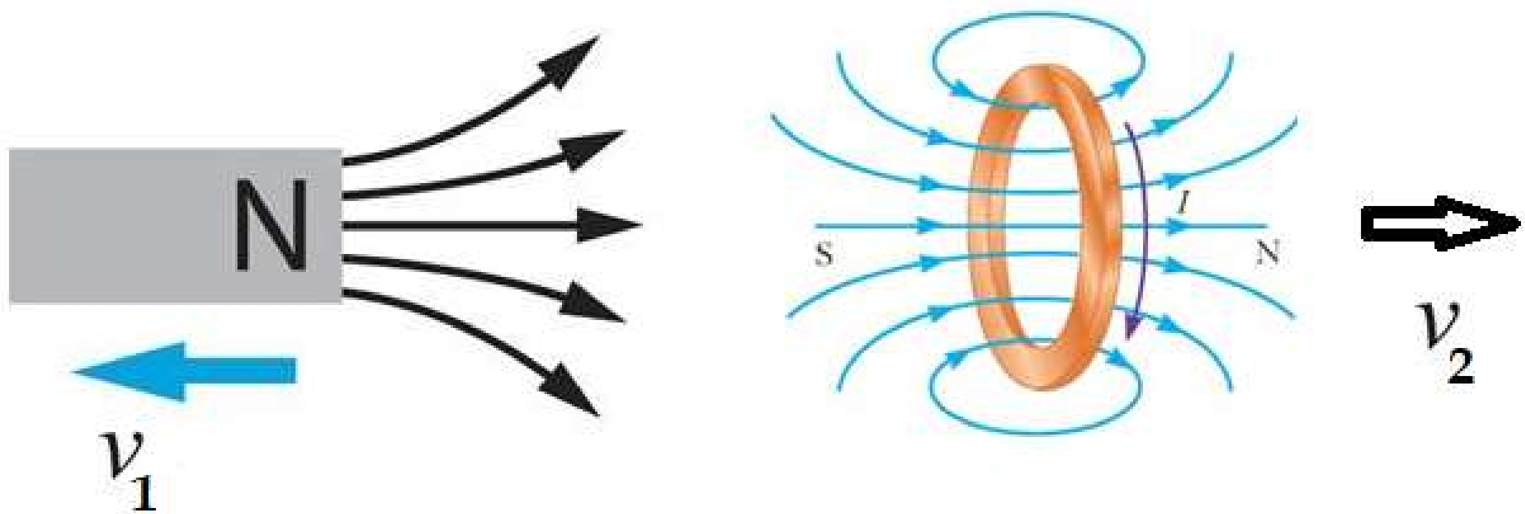
Case 3:

If both are approaching each other

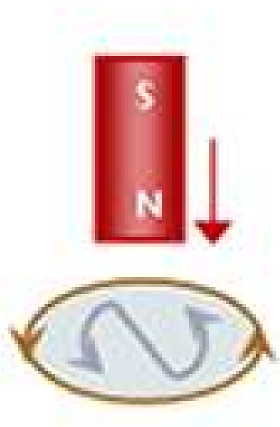


Case 4:

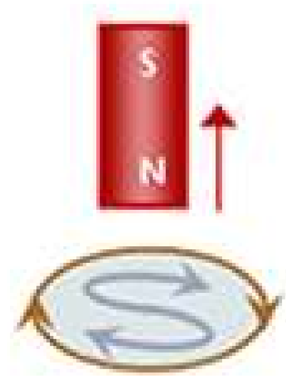
If both are moving away from each other



Lenz’s law – Conservation of energy:



Anti Clock Wise



Clock Wise

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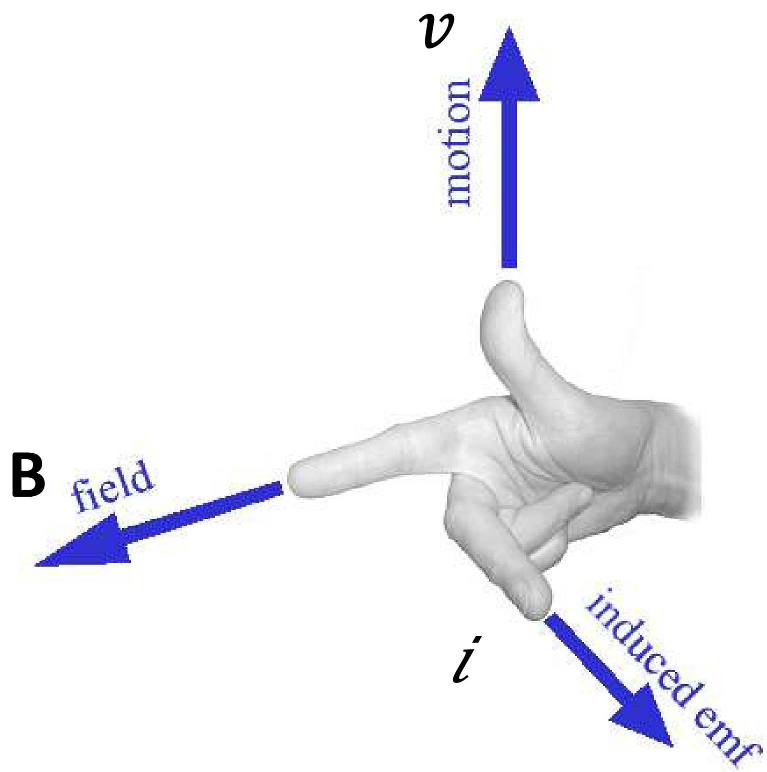
Fleming’s Right Hand Rule:



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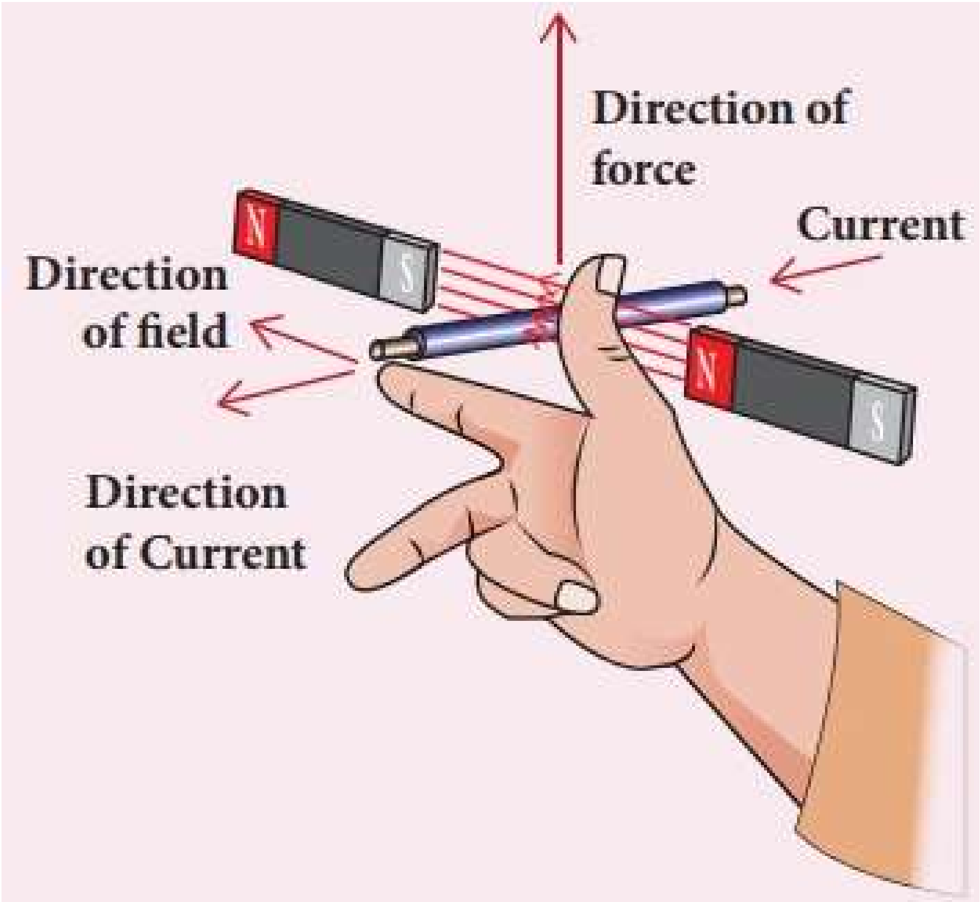

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Motional Emf

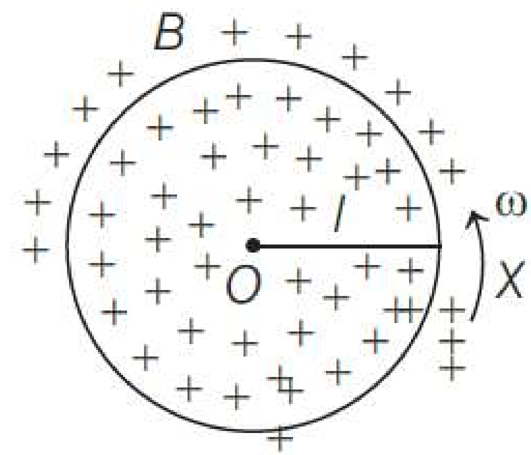
If a rod of length l moves perpendicular to a magnetic field B, with a velocity v, then induced emf produced in it given by $e = Bvl$

If a rectangular coil moves linearly in a field, when coil moves with constant velocity in an uniform magnetic field, flux and induced emf will be zero.

A rod moves at an angle θ with the direction of magnetic field, with velocity v , then

$$e = -Blv \sin\theta$$

If a metallic rod of length l rotates about one of its ends in a plane perpendicular to the magnetic field, then the induced emf produced across, its ends is given by



$$e = \frac{1}{2} B\omega l^2 = BAf$$

where, ω = angular frequency of rotation,

$A = \pi l^2$ = area of circle

and f = frequency of rotation.

If a metallic disc of radius r rotates about its own centre in a plane perpendicular to the magnetic field B, then the induced emf produced between the centre and the edge is given by $e = \frac{1}{2} B\omega r^2 = BAf$

where, ω = angular velocity of rotation,

f = frequency of rotation and $A = \pi r^2$ = area of disc.

The direction of induced current in any conductor can be obtained from Fleming's right hand rule.

Eddy Currents

If a piece of metal is placed in a varying magnetic field or rotated with high speed in an uniform magnetic field, then induced currents set up in the piece are like whirlpool of air, called eddy currents.

The magnitude of eddy currents is given by $I = -\frac{e}{R} = \frac{d\phi/\ dt}{R}$

where, R is the resistance.

Self-Induction

The phenomena of production of induced emf in a circuit due to change in current flowing in its own, is called self-induction.

The magnetic flux linked with a coil, $\phi = LI$

where, $L =$ coefficient of self-induction.

The induced emf in the coil, $e = -L\dfrac{dI}{dt}$

SI unit of self-induction is henry (H) and its dimensional formula is $[ML^2 T^{-2} A^{-2}]$.

Self-inductance of a **long solenoid** is given by normal text,

$$L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 A l$$

where, N = total number of turns in the solenoid,

l = length of the coil,

n = number of turns in the coil

and A = area of cross-section of the coil.

If core of the solenoid is of any other magnetic material, then

$$L = \frac{\mu_0 \mu_r N^2 A}{l}$$

Self-inductance of a **toroid**, $L = \frac{\mu_0 N^2 A}{2\pi r}$

where, r = radius of the toroid.

Energy stored in an inductor, $E = \frac{1}{2} L I^2$.

Mutual Induction

The phenomena of production of induced emf in a circuit due to the change in magnetic flux in its neighbouring circuit, is called mutual induction.

If two coils are coupled with each other, then magnetic flux linked with a coil (secondary coil)

$$\phi = MI$$

where, M is coefficient of mutual induction and I is current flowing through primary coil.

The induced emf in the secondary coil, $e = -M\dfrac{dI}{dt}$

The coefficient of mutual induction depends on geometry of two coils, distance between them and orientation of the two coils.

Mutual inductance of **two long co-axial solenoids** is given by,

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 A l$$

where, N_1 and N_2 are total number of turns in both coils, n_1 and n_2 are number of turns per unit length in coils, A is area of cross-section of coils and l is length of the coils.

Mutual Indutance depends mainly on

i) Geometry of the two circuits.

ii) Mutual inducatance increases with increase in number of turns in the coil.

iii) Mutual inductance decreases with increase in distance between the coils.

iv) Mutual inductance increases with increase in cross sectional area of coils.

v) Mutual inductance increases with permeability of material of the core.

Coefficient of Coupling

Coefficient of coupling of two coils gives a measure of the manner in which the two coils are coupled together.

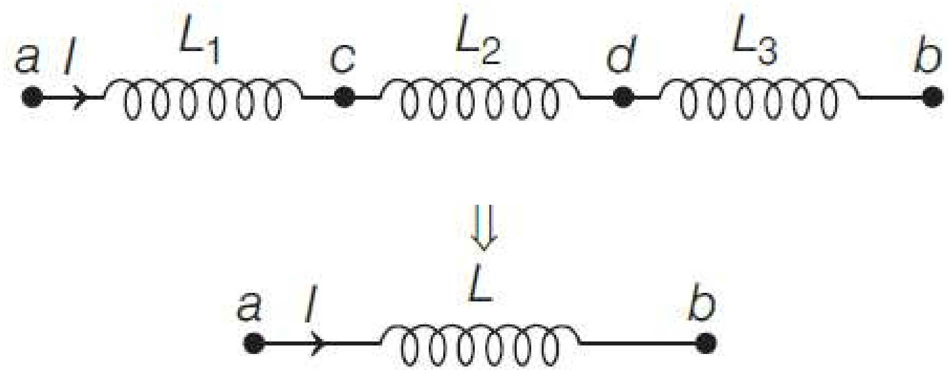
$$K = \sqrt{\frac{M}{L_1 L_2}}$$

where, L_1 and L_2 are coefficients of self-induction of the two coils and M is coefficient of mutual induction of the two coils.

Grouping of Coils

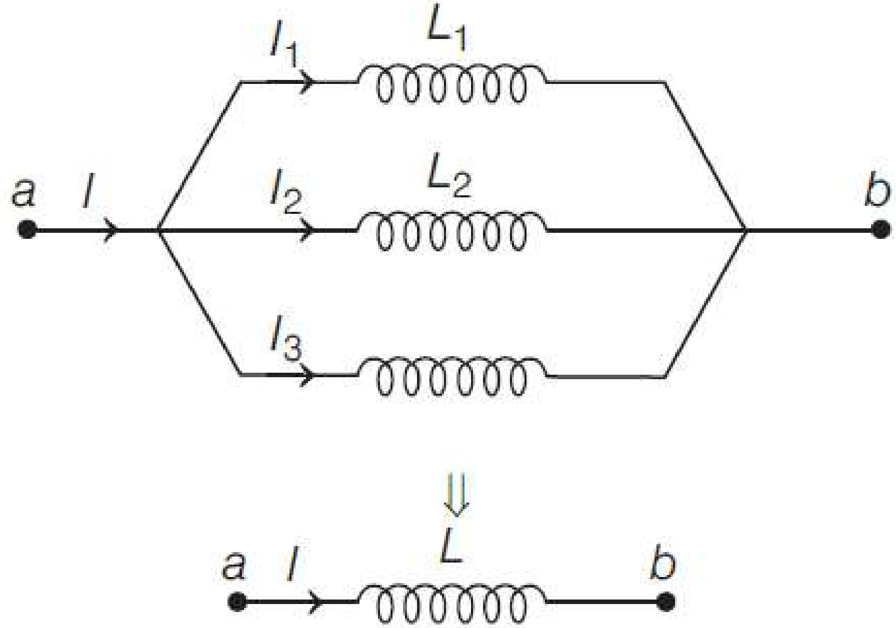
When three coils of inductances L_1, L_2 and L_3 are connected in series and the coefficient of coupling $K = 0$ as in series, then

$$L = L_1 + L_2 + L_3$$



When three coils of inductances L_1, L_2 and L_3 are connected in parallel and the coefficient of coupling $K = 0$ as in parallel, then

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



In series

(a) If current in two coils are in the same direction, then

$$L = L_1 + L_2 + 2M$$

(b) If current in two coils are in opposite directions, then

$$L = L_1 + L_2 - 2M$$

In parallel

(a) If current in two coils are in same direction, then

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

(b) If current in two coils are in opposite directions, then

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

THANK YOU

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