CURRENT ELECTRICITY

FORMULAE

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Electric Current (I)

The rate of flow of charge through any cross-section of a wire w.r.t. time is called electric current flowing through it.

Electric current $(I) = \frac{q}{t}$. Its SI unit is ampere (A).

The conventional direction of electric current is the direction of motion of positive charge.

Direct Current (DC) Its magnitude and direction do not change with time. A cell, battery or DC dynamo are the sources of direct current.

Alternating Current (AC) An electric current whose magnitude changes continuously and changes its direction periodically is called alternating current. AC dynamo is the source of alternating current.

Thermal Velocity of Free Electrons

Free electrons in a metal move randomly with a very high speed of the order of 10^5 ms^{-1}. This speed is called thermal velocity of free electrons. Average thermal velocity of free electrons in any direction remains zero.

Relaxation Time (τ)

The time interval between two successive collisions of electrons with the positive ions in the metallic lattice is defined as relaxation time.

$$\tau = \frac{\text{mean free path}}{\text{rms velocity of electrons}} = \frac{\lambda}{v_{\text{rms}}}$$

Drift Velocity of Free Electrons

When a potential difference is applied across the ends of a conductor, the free electrons in it move with an average velocity opposite to the direction of electric field, which is called drift velocity of free electrons.

Drift velocity $$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$$

where, τ = relaxation time,

e = charge on electron,

E = electric field intensity,

l = length of the conductor,

V = potential difference across the ends of the conductor

and m = mass of the electron.

Relation between electric current and drift velocity is given by

$$v_d = \frac{I}{An\,e}$$

Current Density

The electric current flowing per unit area of cross-section of a conductor is called current density.

$$\text{Current density } (J) = \frac{I}{A} = nev_d$$

Its SI unit is ampere metre^{-2} (Am^{-2}) and dimensional formula is [AT^{-2}].

It is a vector quantity and its direction is in the direction of motion of positive charge or in the direction of flow of current.

Mobility

The drift velocity of electron per unit electric field applied is called mobility of electron.

$$\text{Mobility of electron } (\mu) = \frac{v_d}{E} = \frac{e\tau}{m}$$

Its SI unit is $m^2s^{-1}V^{-1}$ and its dimensional formula is $[M^{-1}T^2A]$.

Ohm’s Law

If physical conditions of a conductor such as temperature remains unchanged, then the electric current (I) flowing through the conductor is directly proportional to the potential difference (V) applied across its ends.

$$I \propto V \quad \text{or} \quad V = IR$$

where, R is the electrical resistance of the conductor

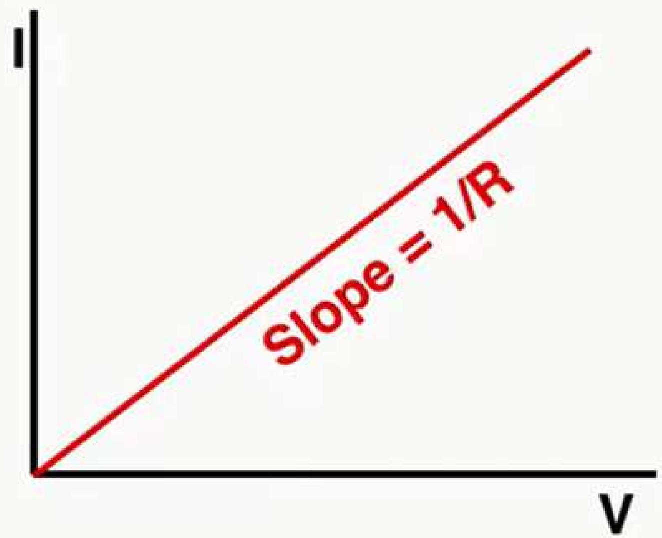
$$\text{and } R = \frac{ml}{Ane^2\tau}.$$

Ohmic vs. Non-Ohmic Resistors

Ohm's Law: $V = IR$

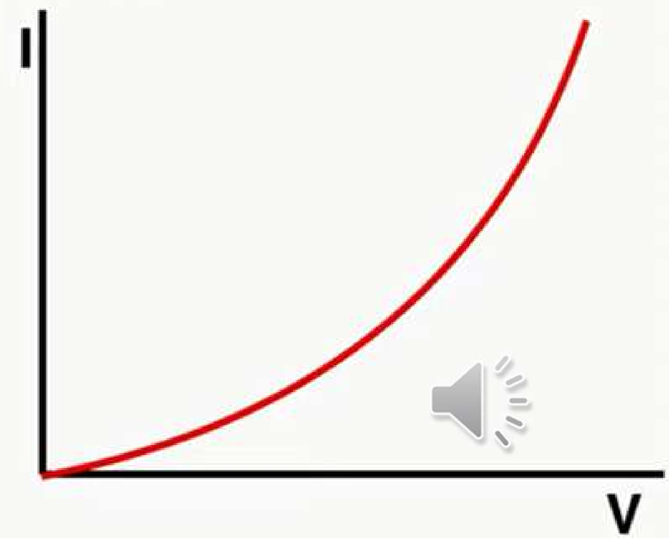
Ohmic Resistors

Resistors that obey Ohm's Law. Any increase in voltage creates a linear increase in the current.



Non-Ohmic Resistors

Resistors that do not obey Ohm's Law. Any increase in voltage creates a non-linear increase in the current. The resistance of the material changes based on the current moving through it.



Ohmic resistor	Non-Ohmic resistor
Ohmic resistor obeys ohm's law i.e., $\frac{V}{I}$ is constant for all values of $V \; or \; I$	Non-ohmic resistor does not obey ohm's law i.e., $\frac{V}{I}$ is not same for all values of $V \; or \; I$.
In Ohmic resistor, $V - I$ graph is linear in nature graph is linear in nature	In non-ohmic resistor, $V - I$ graph is non-linear in nature.
The slope of $V - I$ is same at all values of $V \; or \; I$ at given temperature.	The slope of $V - I$ is different at different values of $V \; or \; I$ at given temperature.

Electrical Resistance

The obstruction offered by any conductor in the path of flow of current is called its electrical resistance.

Electrical resistance, $R = \frac{V}{I}$

Its SI unit is ohm (Ω) and its dimensional formula is $[ML^2T^{-3}A^{-2}]$.

Electrical resistance of a conductor, $R = \frac{\rho l}{A}$

where, l = length of the conductor, A = cross-section area

and ρ = resistivity of the material of the conductor.

Resistivity

Resistivity of a material of a conductor is given by

$$\rho = \frac{m}{ne^2\tau}$$

where, n = number of free electrons per unit volume.

Resistivity is low for metals, more for semiconductors and very high for alloys like nichrome, constantan etc.

Temperature Dependence of Resistivity

Resistivity of a material depend on temperature and nature of the material. It is independent of dimensions of the conductor, *i.e.* length, area of cross-section etc.

Resistivity of metals increases with increase in temperature as

$$\rho_t = \rho_0 (1 + \alpha t)$$

where, ρ_0 and ρ_t are resistivity of metals at 0°C and t°C and α = temperature coefficient of resistivity of the material.

Temperature Dependence of Resistivity

For metals α is positive, for some alloys like nichrome, manganin and constantan, α is positive but very low.

For semiconductors and insulators, α is negative.

Therefore, resistivity of metal increases with increase in temperature.

for semiconductors, it decreases with increase in temperature.

But in the case of alloy, dependence on temperature is weak.

Electrical Conductivity

The reciprocal of resistivity is called electrical conductivity.

Electrical conductivity $(\sigma) = \dfrac{1}{\rho} = \dfrac{l}{RA} = \dfrac{ne^2\tau}{m}$

Its SI units is $\text{ohm}^{-1}\text{m}^{-1}$ or mho m^{-1} or siemen m^{-1}.

Relation between current density (J) and electrical conductivity **(σ)** is given by

$$J = \sigma E$$

where, $E =$ electric field intensity.

Colour coding

Colour	Figure	Multiplier
Black	0	1
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Grey	8	10^8
White	9	10^9

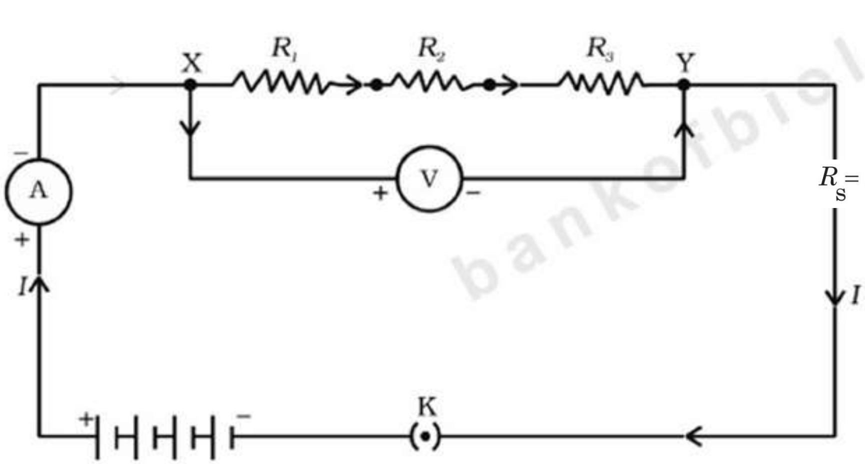
Tolerance power

Colour	Tolerance
Gold	5%
Silver	10%
No colour	20%

RESISTANCE OF A SYSTEM OF RESISTORS

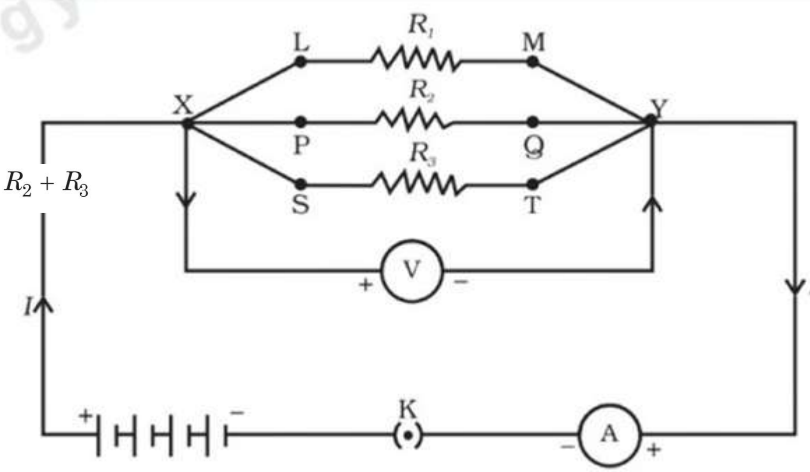
Source: www.bankofbiology.com

- In electrical gadgets, resistors are used in various combinations based on Ohm's law.
- There are 2 methods of joining the resistors: **Resistors in series** & **Resistors in parallel.**



Resistors in series

$R_s = R_1 + R_2 + R_3$



Resistors in parallel

$$R_s = R_1 + R_2 + R_3$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If a skeleton cube is made with 12 equal resistors, each having a resistance R, then the net resistance across

(a) the diagonal of cube $= \frac{5}{6} R$

(b) the diagonal of a face $= \frac{3}{4} R$

(c) along a side $= \frac{7}{12} R$

Electric Cell

An electric cell is a device which converts chemical energy into electrical energy.

Electromotive Force (emf) of a Cell

The energy given by a cell in flowing unit positive charge throughout the circuit completely one time is equal to the emf of a cell.

Emf of a cell $(e) = \dfrac{W}{q}$. Its SI unit is volt.

Internal Resistance of a Cell

The obstruction offered by the electrolyte of a cell in the path of electric current is called internal resistance (r) of the cell. Internal resistance of a cell

(i) increases with increase in concentration of the electrolyte.

(ii) increases with increase in distance between the electrodes.

(iii) decreases with increase in area of electrodes dipped in electrolyte.

Relation between e V and r

$$e = V + Ir$$

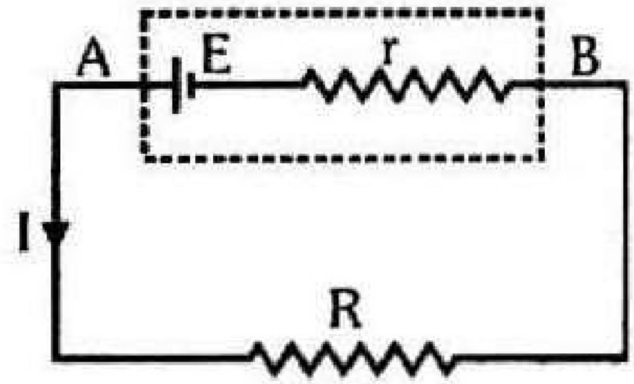
$$r = \left(\frac{e}{V} - 1\right) R$$

If cell is in charging state, then

$$e = V - Ir$$

EMF OF A CELL & ITS INTERNAL RESISTANCE :

If a cell of emf E and internal resistance r be connected with a resistance R the total resistance of the circuit is (R + r) .



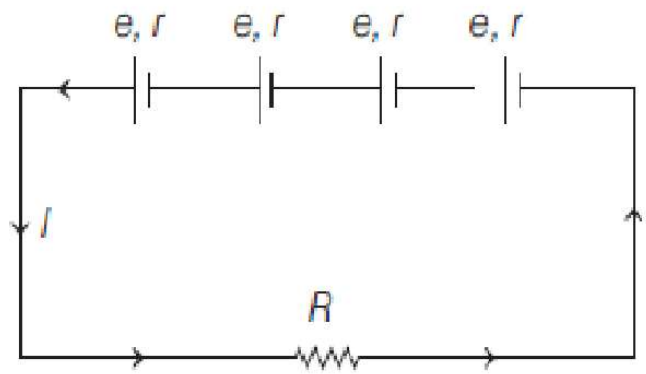
$$I = \frac{E}{R+r} \quad ; V_{AB} = \frac{E}{R+r}$$

where V_{AB} = Terminal voltage of the battery .

If $r \to 0$, cell is Ideal & $V \to E$ & $r = R\left(\frac{E}{V} - 1\right)$

Grouping of Cells

In Series If n cells, each of emf e and internal resistance r are connected in series to a resistance R, then equivalent emf



$$e_{eq} = e_1 + e_2 + \ldots + e_n = ne$$

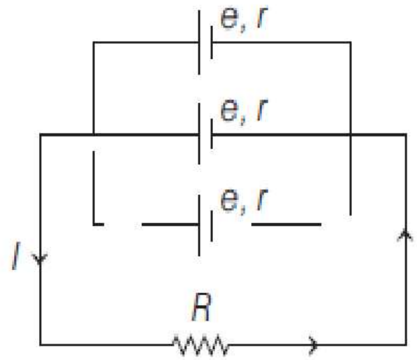
Equivalent internal resistance,

$$r_{eq} = r_1 + r_2 + \ldots + r_n = nr$$

Current in the circuit $I = \dfrac{e_{eq}}{(R + r_{eq})} = \dfrac{ne}{(R + nr)}$

Grouping of Cells

In Parallel If n cells, each of emf e and internal resistance r are connected to in parallel, then equivalent emf, $e_{eq} = e$



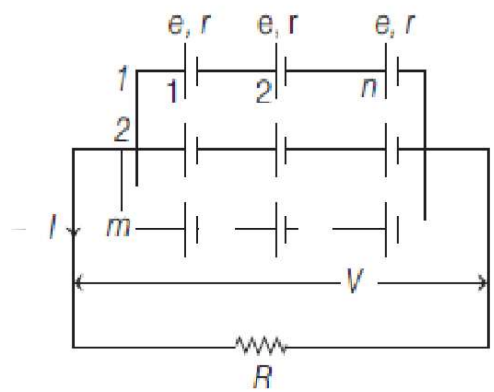
Equivalent internal resistance,

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_1} + \ldots + \frac{1}{r_n} = \frac{n}{r} \quad \text{or} \quad r_{eq} = \frac{r}{n}$$

Current in the circuit, $I = \dfrac{e}{(R + r/\,n)}$

Grouping of Cells

Mixed Grouping of Cells If n cells, each of emf e and internal resistance r are connected in series and such m rows are connected in parallel, then



Equivalent emf, $e_{eq} = ne$

Equivalent internal resistance,

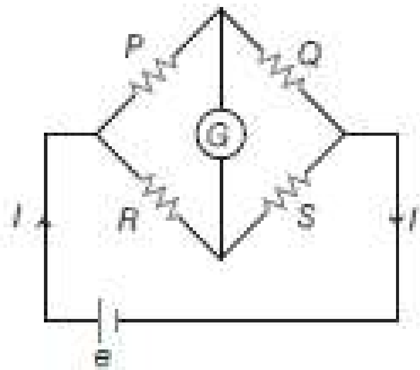
$$r_{eq} = \frac{nr}{m}$$

Current in the circuit, $I = \dfrac{ne}{\left(R + \dfrac{nr}{m}\right)}$ or $I = \dfrac{mne}{mR + nr}$

Wheatstone Bridge

Wheatstone bridge is also known as a **meter bridge** or **slide wire bridge.**

This is an arrangement of four resistances in which one resistance is unknown but rest are known. The Wheatstone bridge is as shown in figure below.



Principle of Wheatstone Bridge

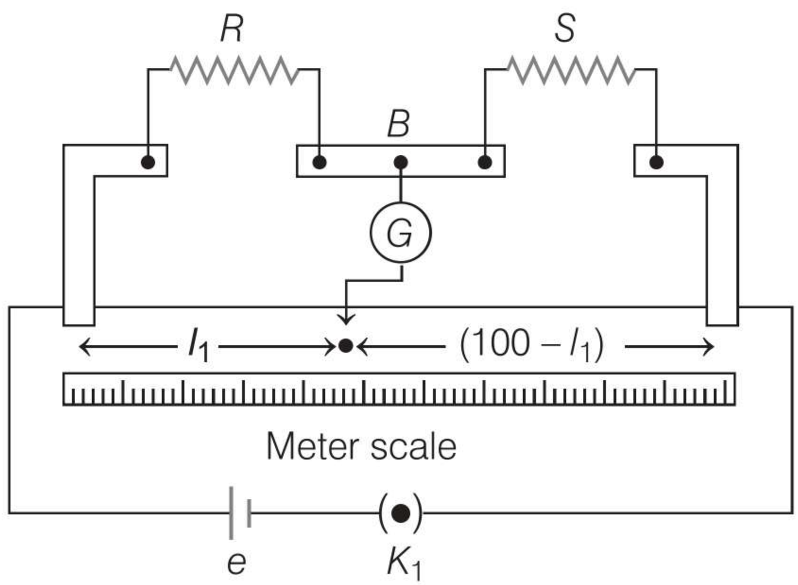
The bridge is said to be balanced when deflection in galvanometer is zero, *i.e.* $I_G = 0$.

Thus, we have the balance condition as $\frac{P}{Q} = \frac{R}{S}$

The value of unknown resistance S can found, as we know the value of P, Q and R. It may be remembered that the bridge is most sensitive, when all the four resistances are of the same order.

Meter Bridge

This is the simplest form of Wheatstone bridge. It is specially useful for comparing resistances more accurately and for measuring an unknown resistance.



At balancing situation of bridge, $\dfrac{R}{S} = \dfrac{l_1}{(100 - l_1)}$

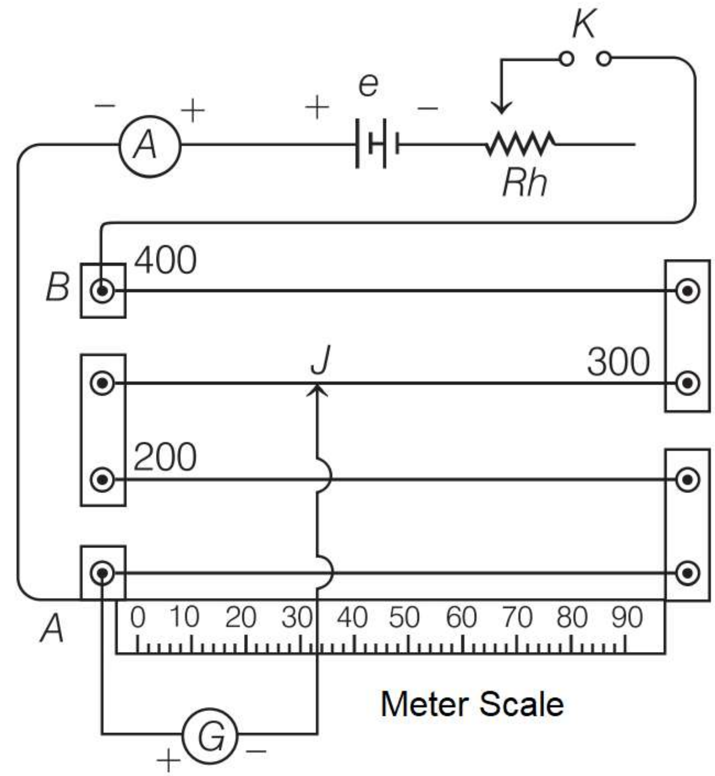
where, l_1 is the length of wire from one end where null point is obtained.

Potentiometer

$$V \propto l$$

$$\Rightarrow \quad V = kl$$

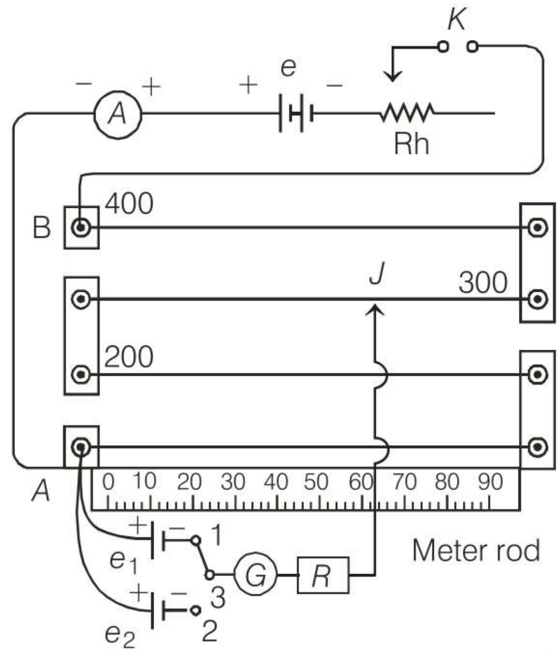
where. k is known as potential gradient.
SI unit of k is Vm^{-1}.



Sensitivity of potentiometer is increased by increasing length of potentiometer wire.

To Compare the emf's of two Cells using Potentiometer

The arrangement of two cells of emfs e_1 and e_2 which are to be compared is shown in the figure below.



If the plug is put in the gap between 1 and 3, we get

$$e_1 = (x\, l_1)\, I$$

where, x = resistance per unit length

Similarly, when the plug is put in the gap between 2 and 3,

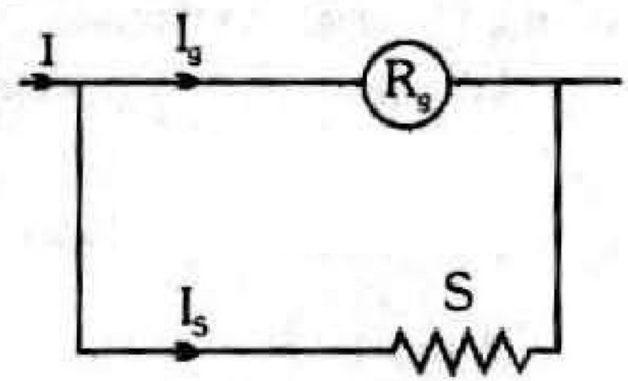
$$e_2 = (x\, l_2)\, I$$

From Eqs. we get

or $$\frac{e_1}{e_2} = \frac{l_1}{l_2}$$

AMMETER

It is a modified form of suspended coil galvanometer, it is used to measure current . A shunt (small resistance) is connected in parallel with galvanometer to convert into ammeter



$$S = \frac{I_g R_g}{I - I_g}$$

where

R_g = galvanometer resistance

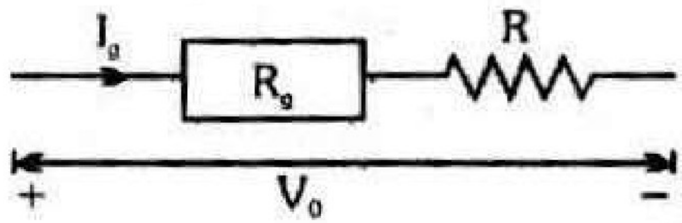
I_g = Maximum current that can flow through the galvanometer .

I = Maximum current that can be measured using the given ammeter .

An ideal ammeter has zero resistance.

VOLTMETER

A high resistance is put in series with galvanometer. It is used to measure potential difference.



$$I_g = \frac{V_o}{R_g + R}\ ;\ R \to \infty\ ,\ \text{Ideal voltmeter}$$

Electric Energy

The energy supplied by any source in maintaining the current in the electric circuit is called electric energy consumed by the electric circuit.

$$\text{Electric energy } (W) = Vq = VIt = I^2Rt = \frac{V^2}{R}t$$

Its SI unit is joule (J) but another unit is watt-hour. The bigger unit of electric energy is kilowatt hour (kWh). It is known as Board of Trade (BOT) unit.

$$\begin{aligned} 1 \text{ kilowatt hour} &= 1000 \text{ watt} \times 1 \text{ hour} \\ &= 1000 \text{ J/s} \times 3600 \text{ s} \\ &= 3.6 \times 10^6 \text{ J} \end{aligned}$$

1 Horse power = 746 watt

The electric energy consumed in kWh is given by

$$W \text{ (in kWh)} = \frac{V \text{ (in volt)} \times I \text{ (in ampere)} \times t \text{ (in hour)}}{1000}$$

Electric Power

The electrical energy produced or consumed per unit time is called electric power.

$$\text{Electric power, } P = VI = I^2R = \frac{V^2}{R}$$

where, V is the potential difference across the conductor, I is current flowing through the conductor and R is the resistance.

Its SI unit is watt (W).

The other units of electric power are kilowatt and horse power.

$$1 \text{ kW} = 1000 \text{ W}$$

$$1 \text{ HP} = 746 \text{ W}$$

Heating Effects of Current

When current I flows through a conductor of resistance R for a time t, then heat generated in it is given by

$$H = I^2Rt \text{ joule} = \frac{I^2Rt}{4.18} \text{ cal} \qquad \text{or}$$

$$H = \frac{V^2}{R} t \text{ joule} = \frac{V^2 t}{4.18\,R} \text{ cal}$$

where, $V =$ potential difference applied across the ends of the conductor.

THANK YOU

JAI HIND

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