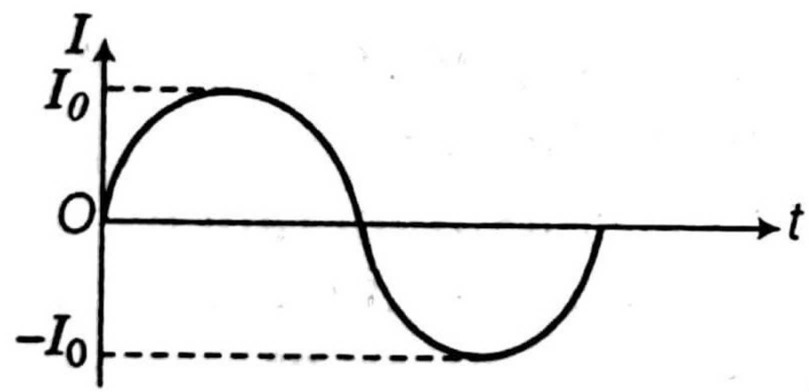
ALTERNATING CURRENTS
FORMULAE

WWW.PHYSICSPOWER.COM

Alternating Current

If the direction of current changes alternatively and its magnitude changes continuously with respect to time, then the current is called alternating current.

$$I = I_0 \sin\omega t$$



where, I = current at any instant t,

I_0 = maximum/peak value of AC,

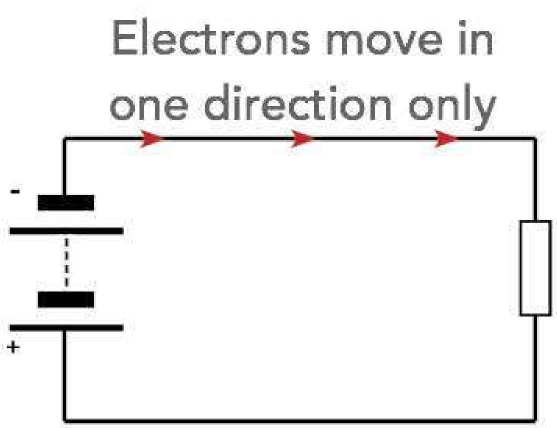
ν = frequency and

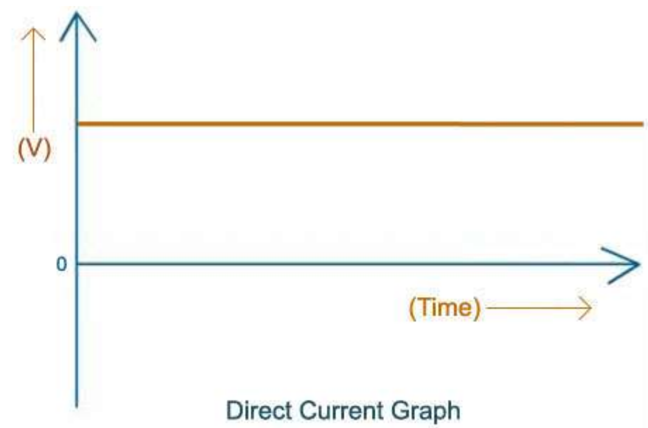
ω = angular frequency.

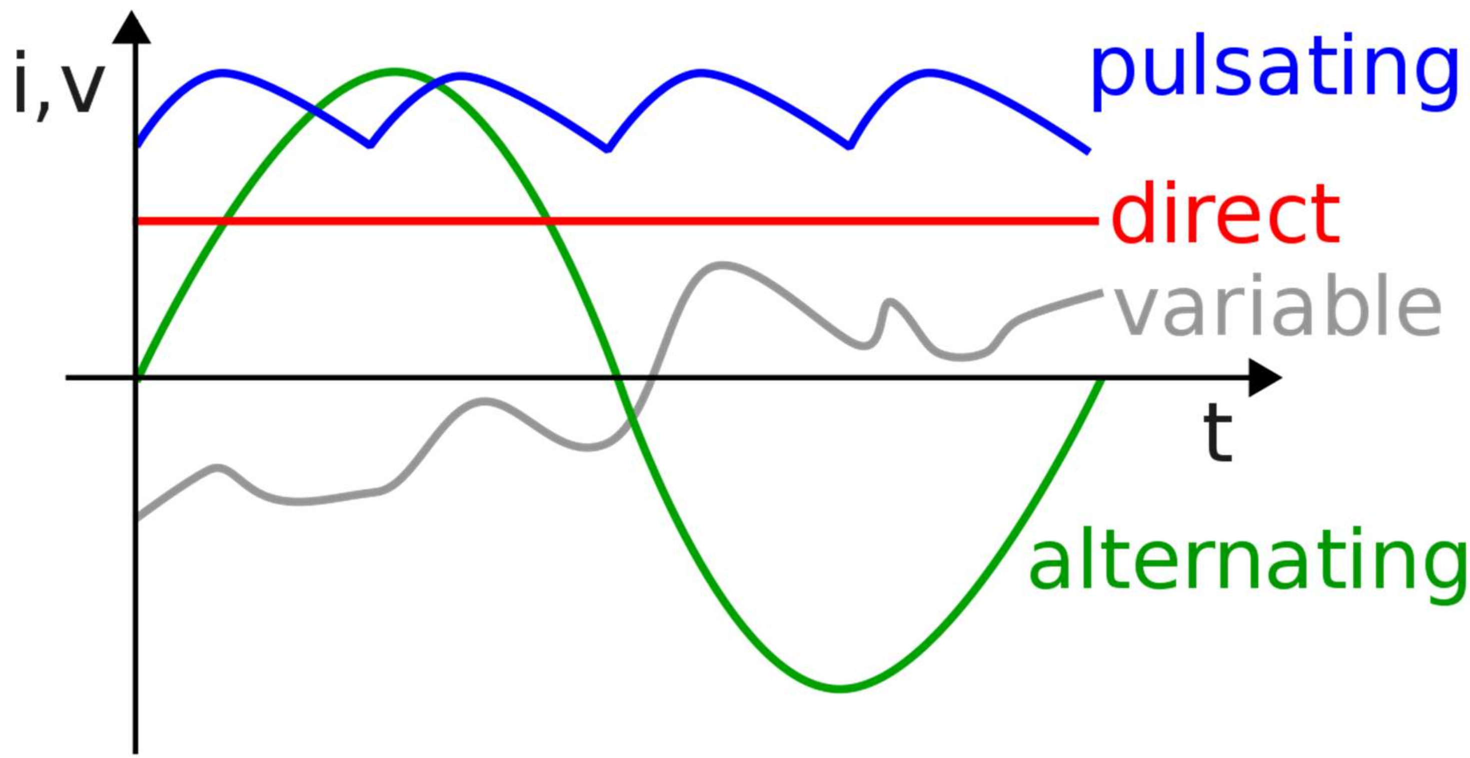
$$E = E_0 \sin\omega t$$

Alternating Current

DC current is defined as a unidirectional flow of electric charge. In DC current, the electrons move from an area of negative charge to an area of positive charge without changing direction.







i,v
pulsating
direct
variable
t
alternating

Advantages of AC over DC

- AC generation is easy and economical
- It can be easily convert into DC with the help of rectifier
- In AC, energy loss is minimum, so it can be transmitted over large distances.

Disadvantages of AC over DC

- AC shock is attractive, while DC shock is repulsive so 220 V AC is more dangerous than 220V DC.
- AC cannot be used in electroplating process because here constant current with constant polarity is needed which is given by DC.

Alternating Current (ac) :

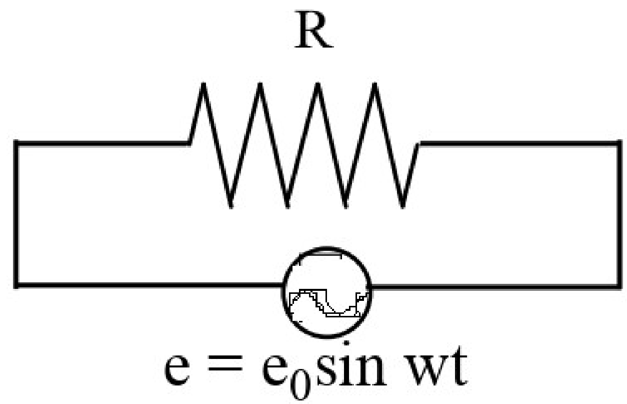
If the current in a circuit changes its direction in every $\frac{T}{2}$ sec (T = Time period in seconds) then the current is called alternating current (**ac**)

The emf of a source which produces sinusoidally varying potential difference across its terminals is given by the equation $e = e_o \sin \omega t$

Where e = instantaneous emf

e_0 = maximum (or) peak emf

Consider an **ac** source connected to a resistor. Then,



$e = e_o \sin \omega t$

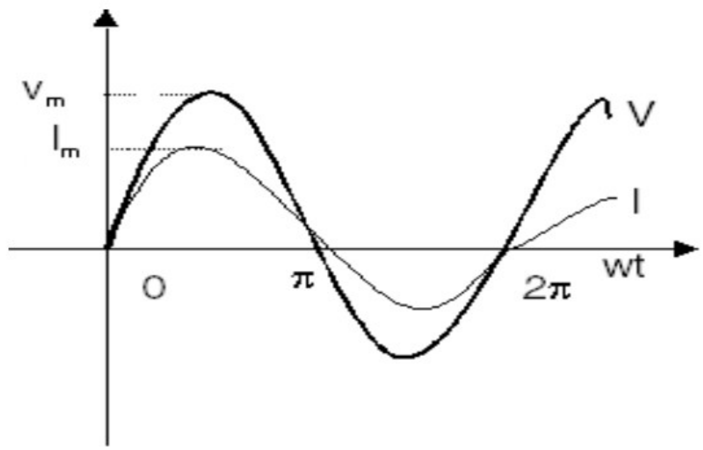
$i = i_o \sin \omega t$

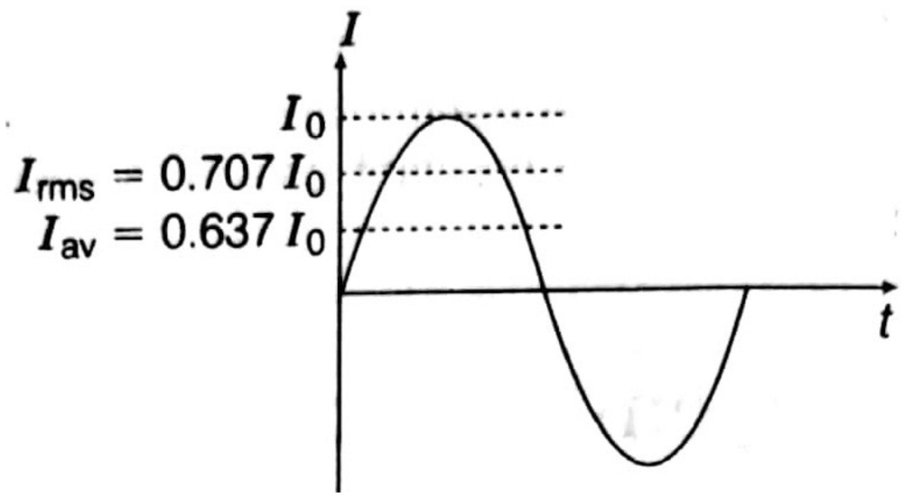
Where i = instantaneous current

i_o = maximum (or) peak current

Both e and i reach maximum and minimum values at the same instant.

the voltage and current are in phase with each other.





Average current over a complete cycle is zero

Its magnitude is obtained by integrating instantaneous value of currents over one half cycle.

$$\text{average value } i_{ave} = \frac{2}{\pi} i_0 = 0.637\ i_o$$

$$\text{similarly } e_{ave} = \frac{2}{\pi} e_0 = 0.637\ e_o$$

Even though average current for complete cycle is zero, power dissipation through the resistor is not zero, as joule heating effect is proportional to i^2

Root mean square (rms) current is the square root of the average of squares of all the instantaneous values of current over one complete cycle.

rms value of current is given by

$$i_{rms} = \frac{i_0}{\sqrt{2}} = 0.707 i_0$$

Similarly rms value of voltage is

$$e_{rms} = \frac{e_0}{\sqrt{2}} = 0.707\ e_0$$

When d.c ammeter is connected to an a.c source it shows zero reading.

The reading given by a.c ammeter is rms current.

Average power loss (P), when a resistor is connected to an a.c source is given by

$$P = V_{rms}^2 / R$$

(or) $P = I_{rms} V_{rms}$

(or) $P = I_{rms}^2 R$

AC CIRCUITS:

Analogous value for resistance for an ideal inductor is $L\omega$. It is called **Inductive reactance** (X_L)

Analogous value for resistance for a capacitor is $1/C\omega$. It is called **capacitive reactance** $(\mathbf{X_C})$

The total resistance offered by the circuit due to capacitor and inductor is called **reactance.**

The total Resistance a circuit offers is called **Impedence (Z).**

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

When only resistance is connected to an a.c source, the current and voltage are in phase. When other circuit elements are also connected, the phase relationship between voltage and current is given by Phasor diagram.

Phasor diagram is only a simple way of finding the Impedence in the circuit and the relationship between current and voltage.

There are two types of phasor diagrams for LCR series circuits

a) Voltage lead b) Current lead

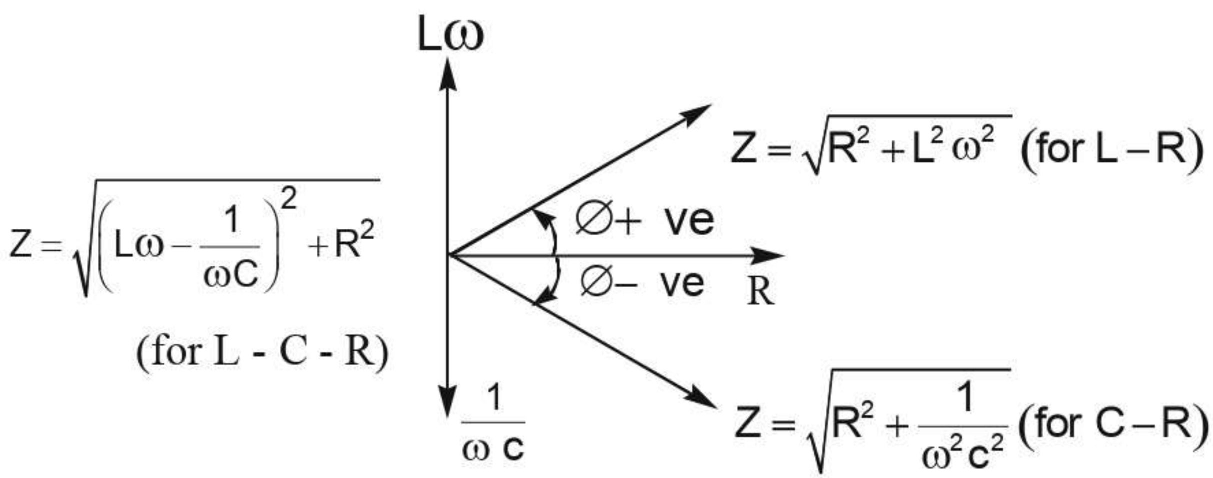
In voltage lead phasor diagram, Resistance is taken along +ve x axis, X_L along + ve y axis, X_C along -ve y axis.

In current lead phasor diagram, Resistance is taken along +ve x axis, X_L along -ve y axis, X_C along +ve y axis.

If emf of an a.c source is given by

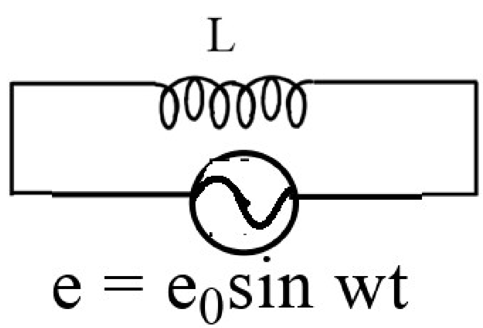
$e = e_0 \sin \omega t$, and current by

$i = i_0 \sin(wt - \phi)$ And if ϕ is the phase difference between current and voltage, then for LCR series circuit **voltage lead phasor diagram** is shown (only impedences are shown)



A.c voltage applied to Inductor :

The instantaneous alternating current is given by $i = i_0 \sin(\omega t - \phi)$

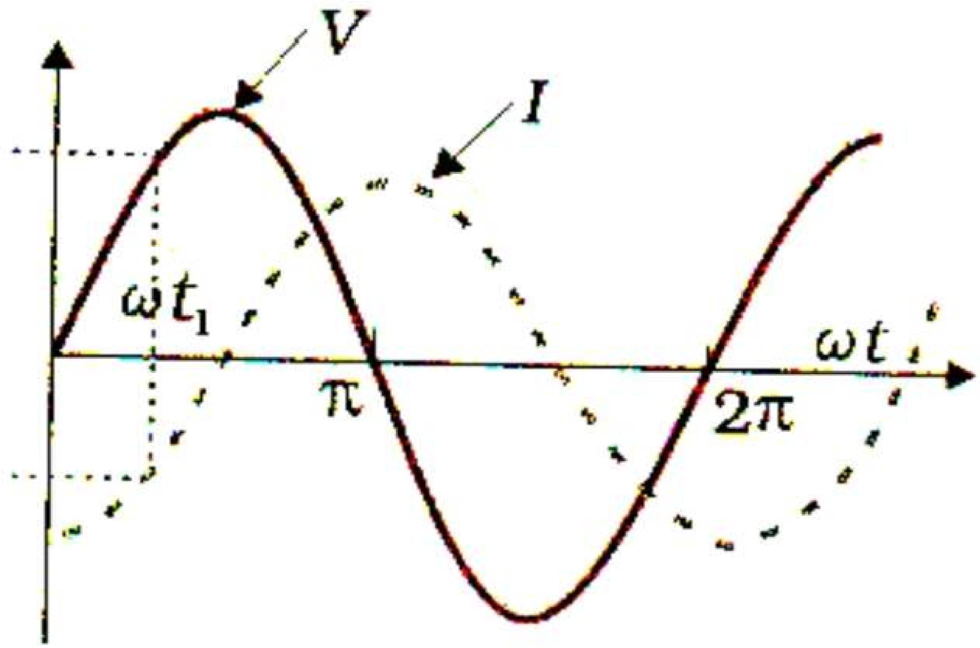


The impedence is given by $Z = L\omega$

The peak or maximum current is given by $i_0 = \frac{e_0}{z}$

The phase difference between emf and current

$$\phi = \frac{\pi}{2}$$



Voltage leads current by $\frac{\pi}{2}$

A.c voltage applied to Capacitor:

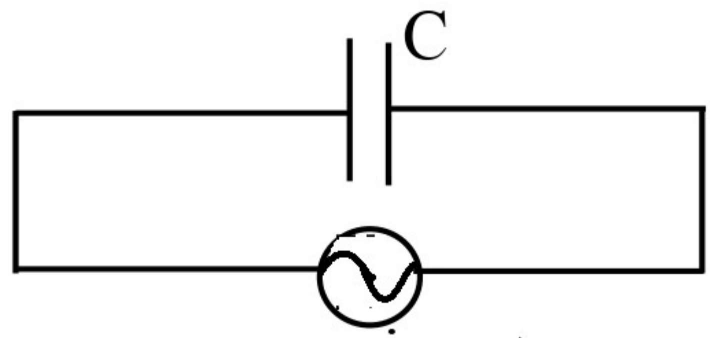
The instantaneous alternating current is given by

$$i = i_0 \sin(\omega t + \phi)$$

The peak or maximum current is given by $i_0 = \dfrac{e_0}{z}$

The impedence is given by

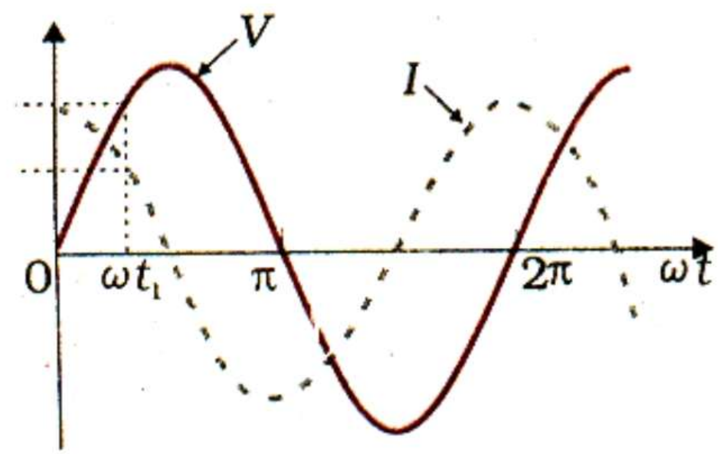
$$Z = \frac{1}{C\omega}$$



$e = e_0 \sin wt$

The phase difference between emf and current

$$\phi = \frac{\pi}{2}$$



Current leads voltage by $\frac{\pi}{2}$

LR AC Circuit:

$$E = E_0 \sin \omega t$$

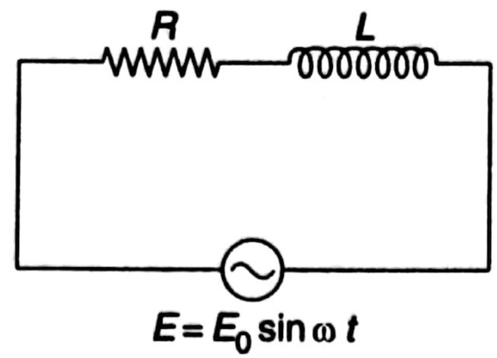
$$i = i_0 \sin(\omega t - \phi)$$

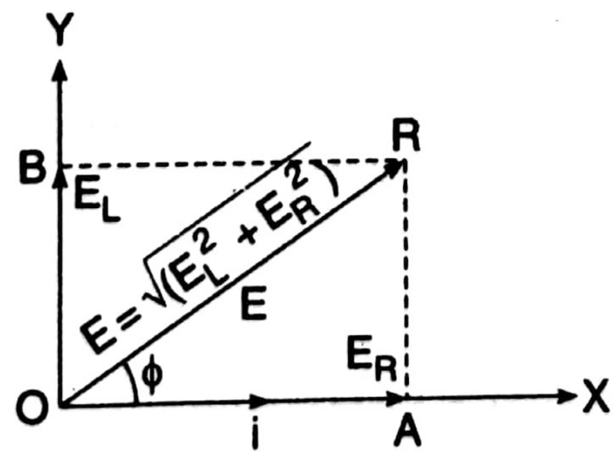
$$E^2 = E_L^2 + E_R^2$$

$E = iZ$, where Z is impedance of the circuit

$$E_R = iR \quad \text{and} \quad E_L = iX_L = i\omega L$$

$$(iZ)^2 = (i\omega L)^2 + (Ri)^2$$





$$Z = \sqrt{(R^2 + \omega^2 L^2)}$$

$$i_0 = \frac{E_0}{\sqrt{[R^2 + \omega^2 L^2]}}$$

$$i = i_0 \sin(\omega t - \phi)$$

$$i = \frac{E_0}{\sqrt{[R_2 + \omega^2 L^2]}} \sin(\omega t - \phi)]$$

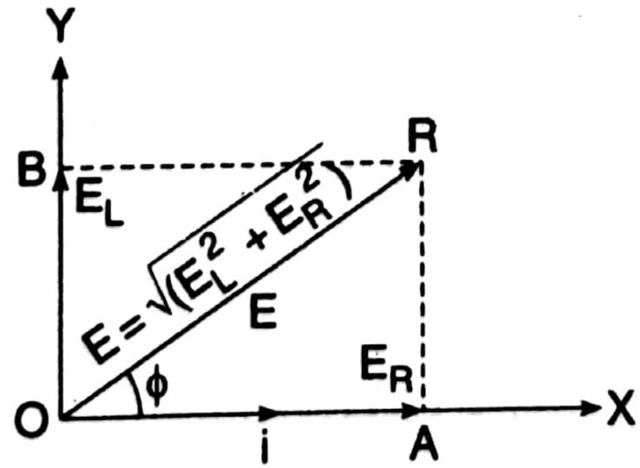
Phase:

$$\tan\phi = \left(\frac{E_L}{E_R}\right)$$

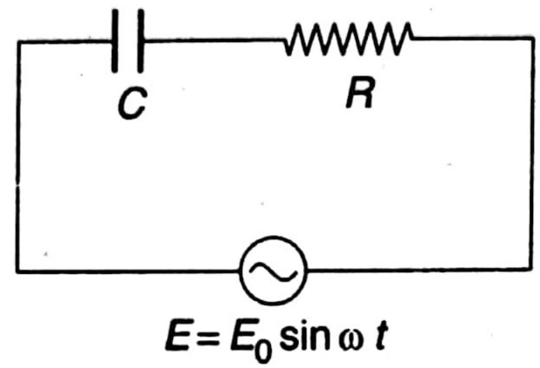
$$E_R = iR \quad \text{and} \quad E_L = iX_L = i\omega L$$

$$= \left(\frac{\omega L}{R}\right)$$

$$\text{or} \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$



CR AC Circuit:

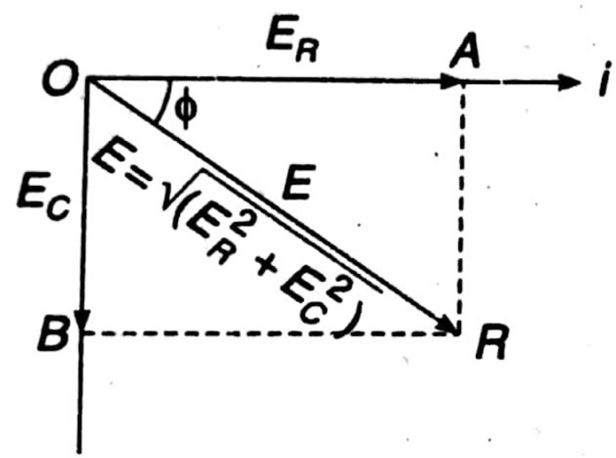


$$i = i_0 \sin(\omega t - \phi)$$

$$E^2 = E_R{}^2 + E_C{}^2$$

where $E = iZ$,

$$E_R = R\,i \quad \text{and} \quad E_C = i\,X_C = \frac{i}{\omega C}$$



$$(iZ)^2 = (R\,i)^2 + (i/\omega C)^2$$

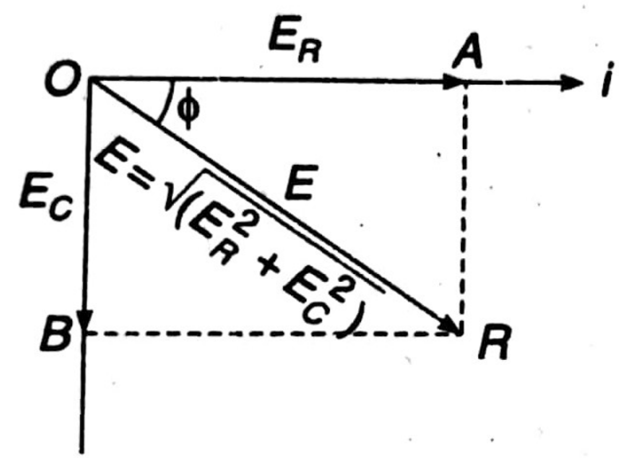
$$(i\,Z)^2 = (R\,i)^2 + (i/\omega\,C)^2$$

$$Z^2 = R^2 + \left(\frac{1}{\omega C}\right)^2 = R^2 + \left(\frac{1}{\omega^2 C^2}\right)$$

$$\therefore \quad Z = \sqrt{\left\{R^2 + \frac{1}{\omega^2 C^2}\right\}}$$

Phase:

$$\tan\phi = \left(\frac{E_C}{E_R}\right)$$



$$E_R = R\,i \quad \text{and} \quad E_C = i\,X_C = \frac{i}{\omega C}$$

$$\tan\phi = \frac{\left(\frac{1}{\omega C}\right)i}{R\,i}$$

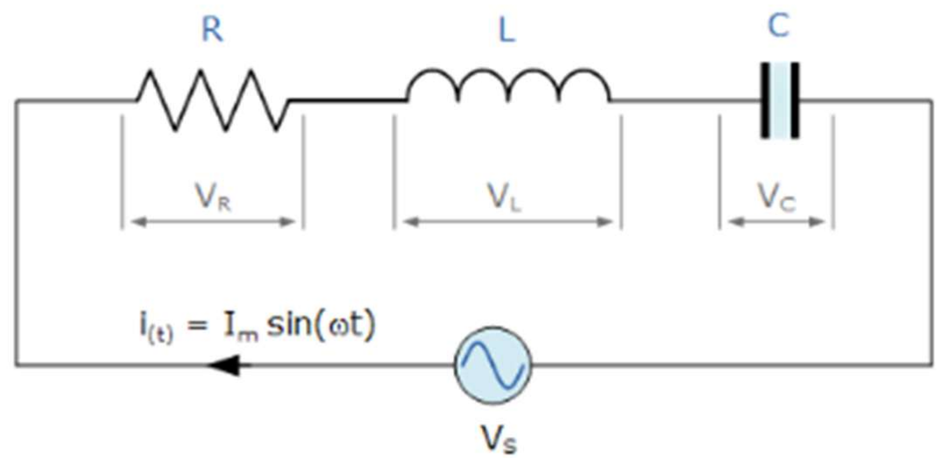
$$\phi = \tan^{-1}\left\{\frac{1/\omega C}{R}\right\}$$

A.c voltage applied across L-C-R series combination

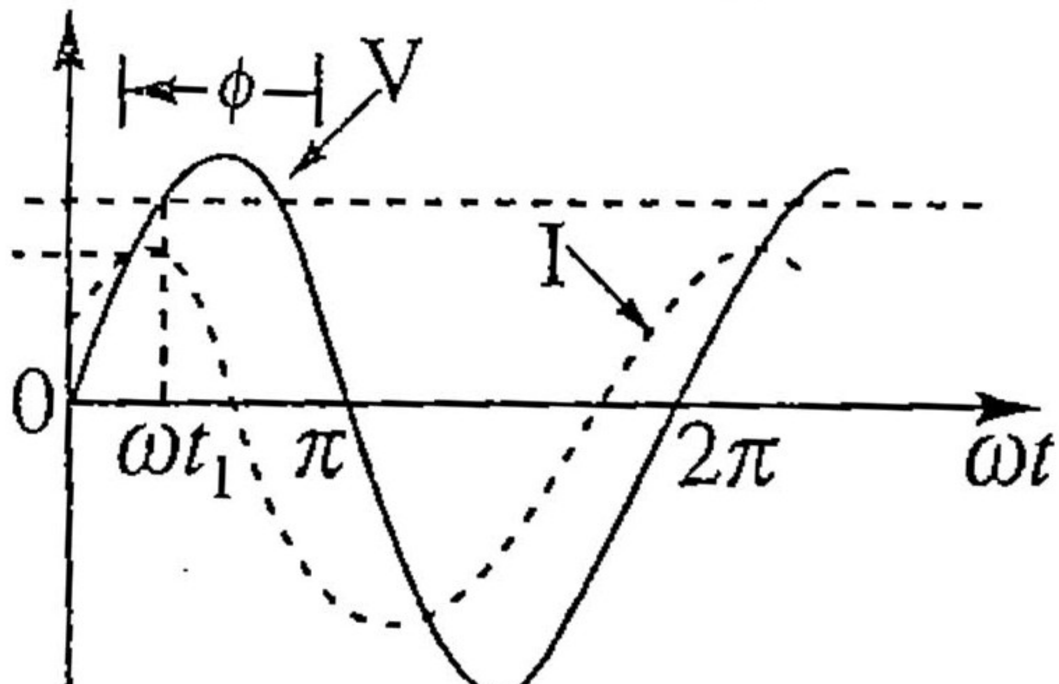
The instantaneous alternating current is given by $i = i_0 \sin(\omega t \pm \phi)$

The impedence is given by

$$z = \sqrt{\left(L\omega - \frac{1}{\omega C}\right)^2 + R^2}$$



The phase difference between emf and current is given by $\phi = \tan^{-1}\dfrac{\left(L\omega - \dfrac{1}{\omega C}\right)}{R}$.



case-i If $L\omega > \dfrac{1}{\omega C}$; ϕ is positive Voltage leads current by ϕ. Circuit is predominantly **Inductive**

case-ii If $\dfrac{1}{\omega C} > L\omega$; ϕ is negative

Current leads voltage by ϕ

Circuit is predominantly **capacitive**

case-iii If $L\omega = \frac{1}{\omega c}$; ϕ is zero . Voltage and current are inphase. This condition is called **Resonance**

at resonance $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\boxed{n_0 = \frac{1}{2\pi\sqrt{LC}}}$$

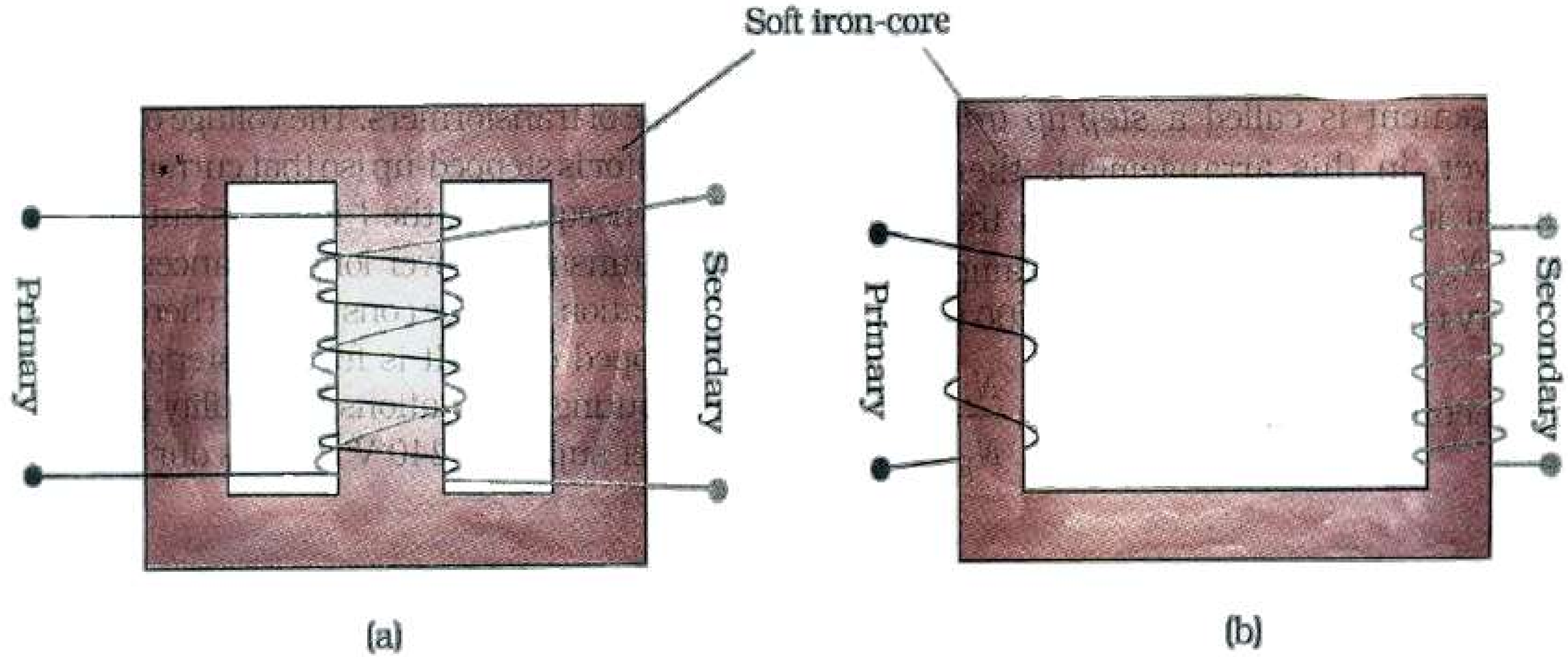
This frequency is called resonant frequency At resonant frequency, the current amplitude (i_0) is maximum

Resonant circuits are used in tuning mechanism of a radio (or) a TV set and musical instruments.

In LCR circuit, the total applied voltage (V) across L, C, R is given by

$$V = \sqrt{(V_L - V_C)^2 + V_R^2}$$

Transformer:



Transformer:

i) Transformer is used to transform an alternating voltage from one coil to another.

ii) An ideal transformer is one in which the primary has negligible resistances and all the flux in the core links both primary and secondary windings.

iii) If ϕ be the flux in each turn in the core and if a voltage V_P is applied in primary, then emf induced in secondary with N_s turns is

$e_s = -N_s \frac{d\phi}{dt}$ and emf induced in primary with

N_P turns is $e_P = -N_P \frac{d\phi}{dt}$ But $e_P = V_P$

(Since it is assumed that primary has zero resistance)

If the secondary is an open circuit, (or) if current taken from secondary is small, then $e_s = V_s$.

where V_s is the voltage across secondary.

$$\therefore \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

transformer is assumed to be 100% efficient (no energy loss) then,

Input power = Out put power

$I_P V_P = I_S V_S$

$$\boxed{\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}} \qquad V_S = \left(\frac{N_S}{N_P}\right) V_P$$

If $N_S > N_P$ voltage is stepped up, then the transformer is called step - up transormer.

If $N_S < N_P$ voltage is stepped down, then the transformer is called step - down transformer.

In step - up transformer, $V_S > V_P$ and $I_S < I_P$

In step - down transformer, $V_S < V_P$ and $I_S > I_P$.

Frequency of input a.c is equal to frequency of output a.c.

The core of the transformer must be made of a material with high magnetic permeability.

In step-up transformers primary is made of thick insulated copper wire and secondary is made of a thin wire.

In step-down transformer primary is made of thin insulated copper wire and secondary is made of a thick wire.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}}$$

$$\text{Percentage of efficiency} = \frac{output\ power}{Input\ power} \times 100$$

For an ideal transformer efficiency is 100%. But due to energy losses in transformers the efficiency varies between 90% and 99%.

Small energy losses are due to following reasons.

Flux leakage : This is due to poor design of the core (or) the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

Small energy losses are due to following reasons.

Resistance of the winding : The wire used for the windings has some resistance. Therefore energy is lost due to heat produced in the wire (I^2R). This loss is minimised by using thick wire.

Small energy losses are due to following reasons.

Eddy currents : The alternating magnetic flux produces eddy currents in the iron core and causes heating. This effect is reduced by having a laminated core.

Small energy losses are due to following reasons.

Hysteresis : The magnetisation of the core is repeatedly reversed by the alternating magnetic field in core appears as heat. This loss is kept to a minimum by using a magnetic material which has a low hysteresis loss.

THANK YOU

JAI HIND