ATOMS

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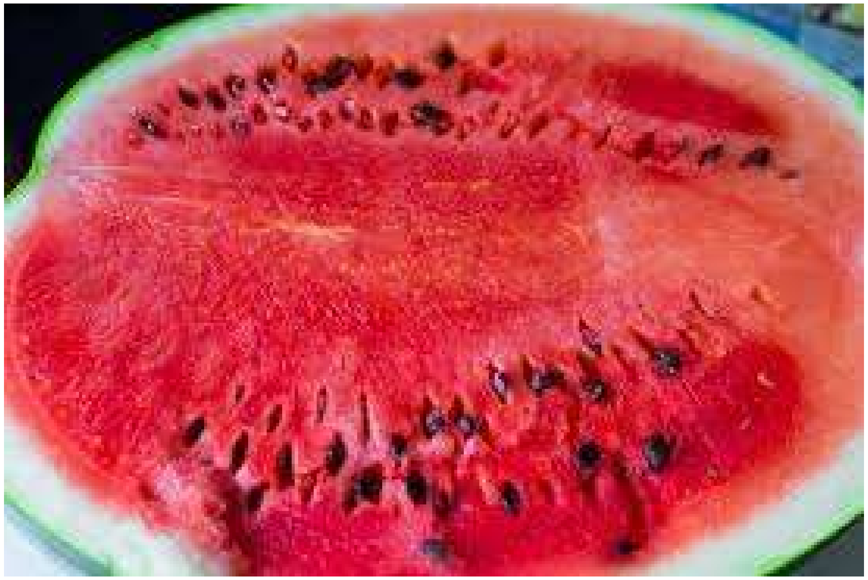
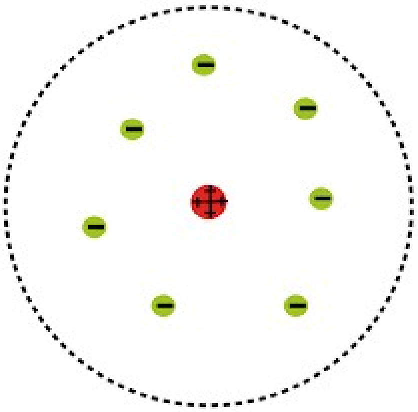
To explain the atomic structure Several theories are...

Those are,

J J THOMSON - Plum Pudding model



Rutherford Atomic Model

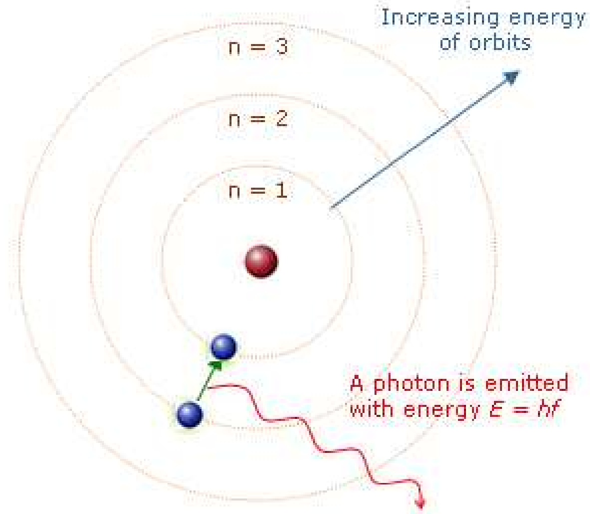


Alpha Scattering experiment

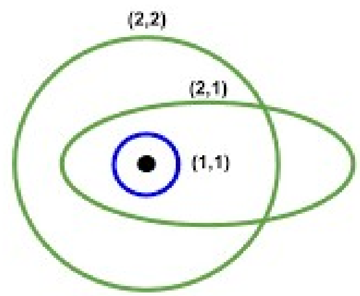
- **Nucleus –**
- **Circular Orbits -**

Bohr Atomic Model

- Stationary Orbits
- Angular momentum
- Fixed finite energy states



Somerfield's Relativistic Model



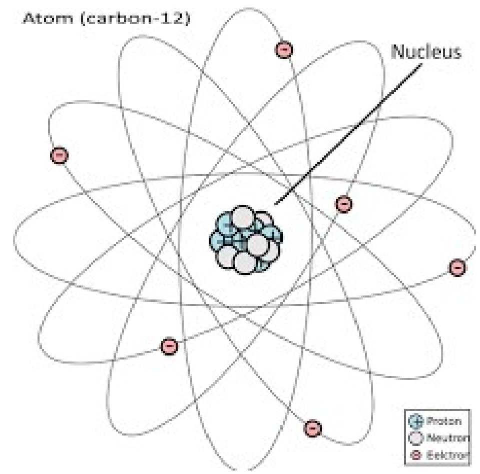
- Elliptical orbits
- Fine structure of spectral lines

(One spectral line composed of more number of spectral lines)

Vector Atom Model

This model is the extension of Rutherford-Bohr-Somerfield

Atomic Models



JJ THOMSON - **Plum Pudding model**

According to his theory

- Total positive charge - reddish part in the watermelon
- negative charge – electrons – nuts in the watermelon

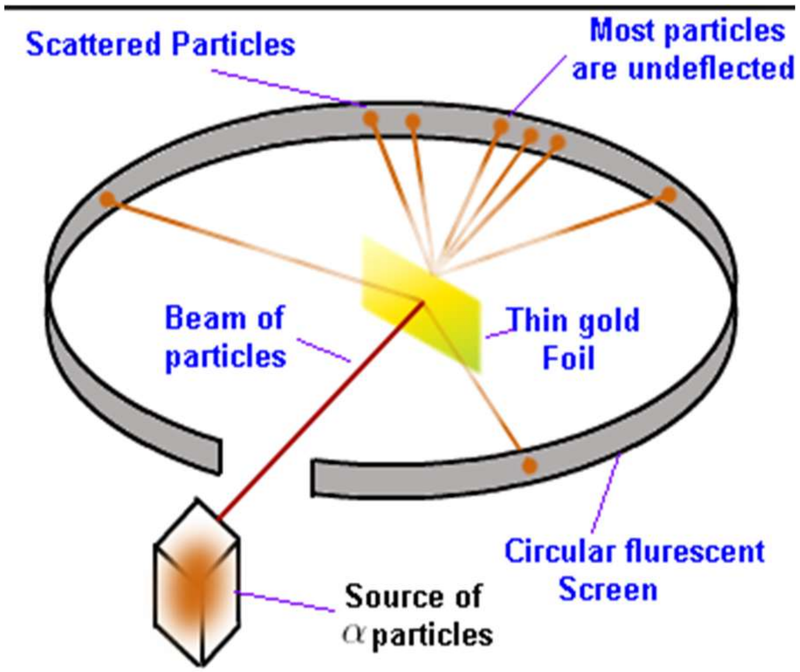
RUTHERFORD

ALPHA SCATTERING EXPERIMENT

Experimental Arrangement:

- Image source www.zigya.com

- Alpha particles source – **Bi^{214}**
- 0.2 micrometer thick gold foil
- ZnS Screen



Graphical Analysis

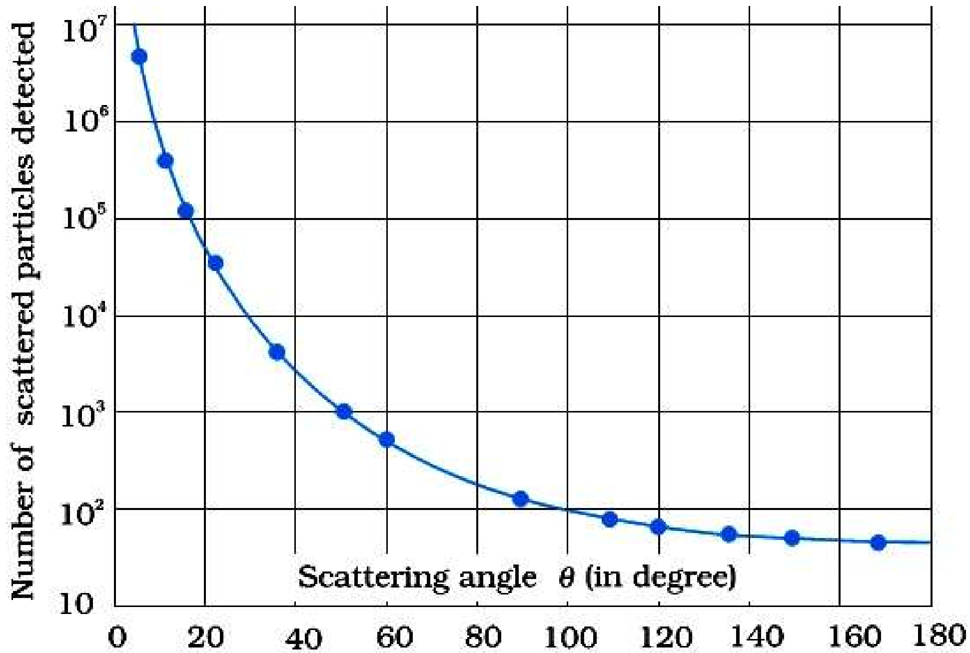


Image source: physicsopenlab.org

Observations

- Deflection of alpha particles is very less
- Only 0.14% of alpha particles scattered by more than one degree
- Only one out of 8000 alpha particles deflected by more than 90 degree

(1 D Head on collision)

Conclusions:

- **Atom is almost empty**
- **Entire mass of the atom is concentrated at the centre of the atom called nucleus**
- **Size of the nucleus is very small and is the order of one fermi**
- **Size of the atom is 100000 times more than the nucleus**

Rutherford Atomic Model / Planetary Model

- In every atom, total heavy positive charge located at the centre of the atom called nucleus.
- Size of the atom is the order of 1 A^0 and size of the nucleus is the order of 1 fermi (10^{-15} m)

- Nucleus is surrounded by electrons
- Electrons revolve around the nucleus in circular orbits

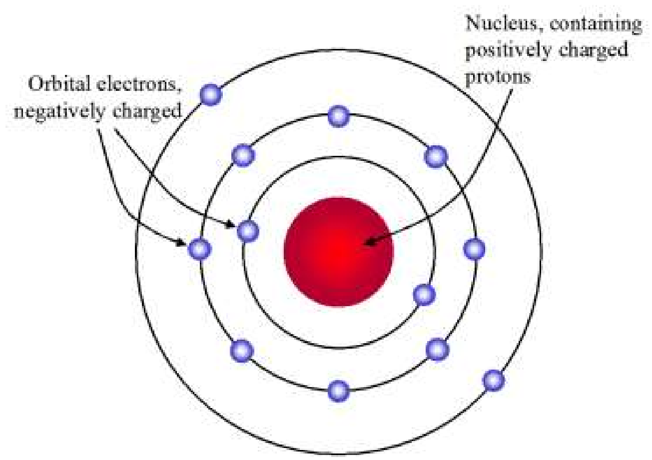


Image source: stamfordnuclearphysics.weebly.com/

www.physicspower.com

Theory:
Distance of Closest Approach

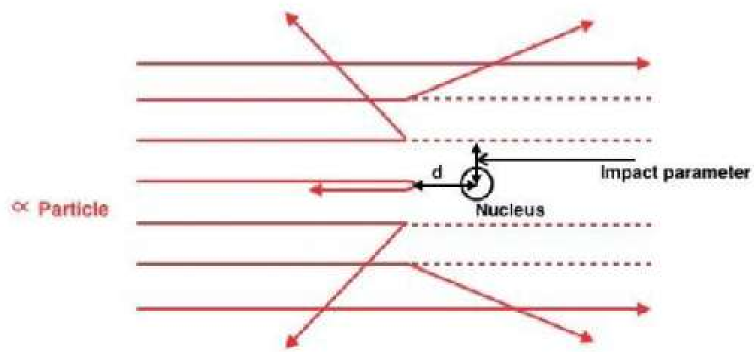


Image source: meritnation.com

According to conservation of energy **KE = PE**

Kinetic energy of alpha particle = Electrostatic potential energy between nucleus and alpha particle

$$KE_{(\alpha)} = P.E_{(N,\alpha)}$$

$$\frac{1}{2}mv_0^2 = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{b}$$

Where v_0 - initial velocity of α-particle

q_1 - charge of the nucleus $= Ze$

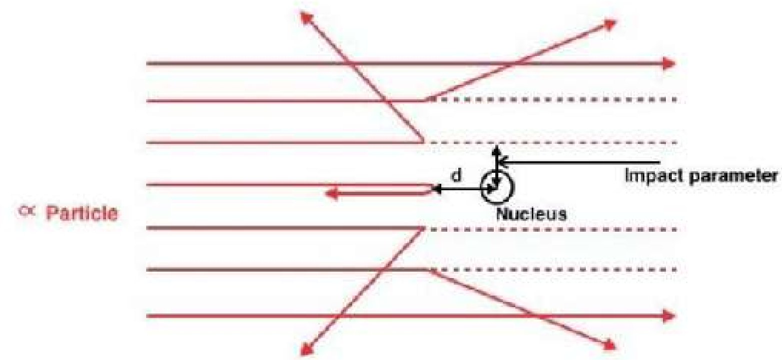
q_2 - charge of α-particle $= 2e$

b - distance of closest approach

$$\therefore \frac{1}{2}mv_0^2 = \frac{1}{4\pi\epsilon_0}\cdot\frac{(Ze)(2e)}{b}$$

$$\Rightarrow \boxed{b = \frac{Ze^2}{\pi\epsilon_0 m v_0^2}}$$

$$\text{or} \quad \boxed{b = \frac{2Ze^2}{4\pi\epsilon_0 \, K.E}}$$



Impact Parameter

Perpendicular distance between initial direction of alpha particle and the nucleus

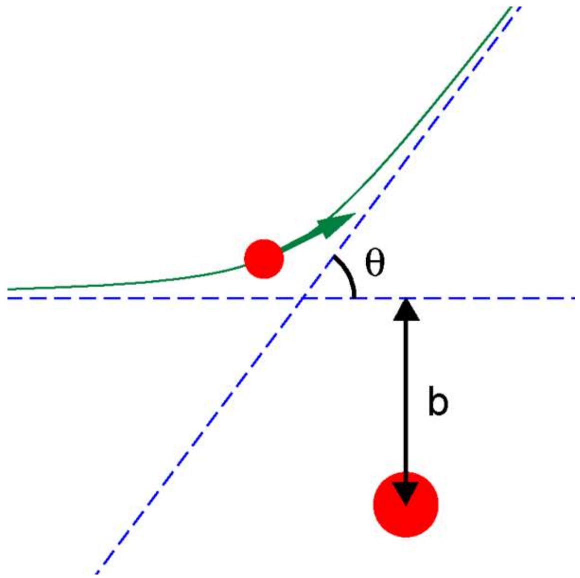


Image source: wikimedia.org

Scattering angle

Angle between initial and final directions of alpha particle.

Rutherford Scattering Formula:

Number of alpha particles scattered, N

$$N(\theta) = \frac{Z^2 e^4 \, n t Q}{4 r^2 \, K.E^2 \sin^4(\theta/2)}$$

$$\Rightarrow N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

Where, n – number of atoms per unit volume of gold foil

t – thickness of the gold foil

Q – probable number of particles on the surface of the target per unit area

r – distance between gold foil and the screen

Drawbacks:

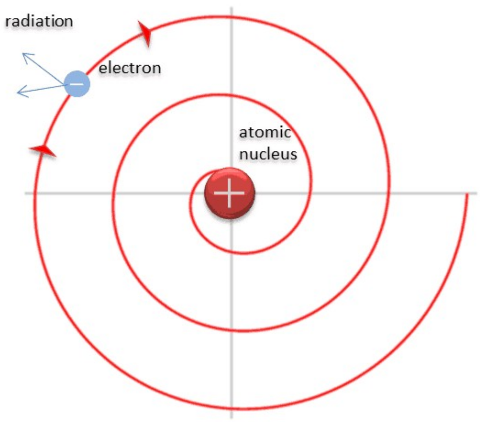


Image source: www.nuclear-power.net

- Electrons are moving –
  must radiate energy
- Due to loss of energy –
  electron radius should decrease
  and fall in the nucleus

Thus atom cannot be a stable.

Rutherford was unable to explain the stability of the atom

BOHR ATOMIC MODEL

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BOHR ATOMIC MODEL

POSTULATES

- Nucleus
- Stationary Orbits
- Angular momentum quantised

- Energy emitted by the atom $E = h\nu$

Stationary Orbits

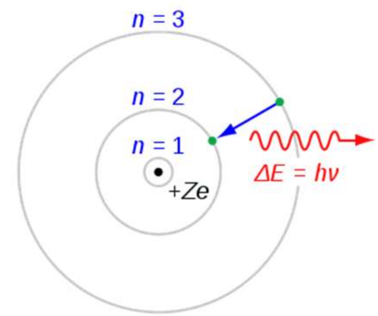


Image source: en.wikipedia.org

- ❑ Each atom has certain definite stable energy states/orbits in which electrons revolve continuously.
- ❑ As long as the electrons are revolving in these energy states, electrons will not lose energy and gain energy.

Angular momentum quantised

- Fixed finite set of energy states are allowed for with angular momentum quantised

$$L = \frac{nh}{2\pi}$$

$$mvr = \frac{nh}{2\pi}$$

Where n = 1,2,3,… or K,L,M,N,…
called **Principal Quantum number**

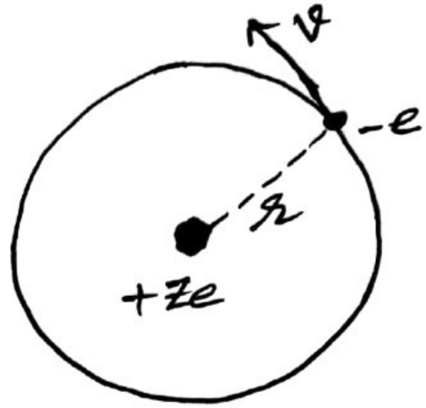
Energy Emission

Energy emitted by the atom E = hν

$$\Delta E = E_i - E_f = h\nu \qquad E_i > E_f$$

BOHR THEORY

Radius of the electron in an orbit:



Form his postulate, angular momentum

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi mr} \rightarrow (1)$$

$$F_{centrifugal} = \frac{mv^2}{r} \qquad F_{coulomic} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(Ze)(e)}{r^2}$$

$$F_{centrifugal} = F_{coulombic}$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\cdot\frac{(Ze)(e)}{r^2}$$

$$\Rightarrow r = \frac{Ze^2}{4\pi\epsilon_0 mv^2} \rightarrow (2)$$

$$\Rightarrow r = \frac{Ze^2}{4\pi\epsilon_0 m\left[\frac{nh}{2\pi mr}\right]^2}$$

$$\Rightarrow r = \frac{Ze^2}{4\pi\epsilon_0 m \frac{n^2h^2}{4\pi^2 m^2 r^2}}$$

$$\Rightarrow r = \frac{Ze^2}{4\pi\epsilon_0 m \frac{n^2h^2}{4\pi^2 m^2 r^2}}$$

$$\Rightarrow r = \frac{Ze^2 \pi m r^2}{\epsilon_0 n^2 h^2}$$

$$\Rightarrow r = \frac{n^2 h^2 \epsilon_0}{Ze^2 \pi m}$$

In general, $r_n = \dfrac{n^2 h^2 \epsilon_0}{Ze^2 \pi m} \rightarrow (3)$

$r_n \propto n^2$ $\quad \boxed{\dfrac{r_1}{r_2} = \dfrac{n_1^2}{n_2^2}}$ $\qquad r_n \propto \dfrac{1}{Z} \Rightarrow \boxed{\dfrac{r_1}{r_2} = \dfrac{Z_2}{Z_1}}$

For hydrozen atom, $Z = 1$

$$\therefore r_n = \frac{n^2 h^2 \epsilon_0}{e^2 \pi m} \rightarrow (*)$$

Planck's constant $h = 6.625 \times 10^{-34}$ Js

Permittivity of the free space $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

Mass of the electron $m = 9.1 \times 10^{-31}$ kg

Charge of the electron $e = 1.6 \times 10^{-19}$ C

$$\boxed{r_n = 0.529\ n^2}\ A^\circ \rightarrow (4)$$

Velocity of the electron

$$v = \frac{nh}{2\pi m r} \rightarrow (1) \qquad r_n = \frac{n^2 h^2 \epsilon_0}{Ze^2 \pi m} \rightarrow (3)$$

$$v = \frac{nh}{2\pi m \left[\frac{n^2 h^2 \epsilon_0}{Ze^2 \pi m}\right]}$$

$$\Rightarrow \quad v = \frac{Ze^2}{2nh\epsilon_0}$$

In general, $v_n = \frac{Ze^2}{2nh\epsilon_0} \rightarrow (5)$

$$v_n = \frac{Ze^2}{2nh\epsilon_0} \rightarrow (5)$$

$$v_n \propto Z$$

$$v_n \propto \frac{1}{n}$$

For hydrogen atom, $Z = 1$

$$\therefore \quad v_n = \frac{e^2}{2nh\epsilon_0} \rightarrow (**)$$

$$\Rightarrow v_n = \frac{e^2 c}{2nh\epsilon_0 c}$$

If $n = 1$, $\frac{e^2}{2h\epsilon_0 c} = \frac{1}{137}$

$$\therefore \quad v_{n=1} = \frac{c}{137}$$

Energy of electrons in Stationary Orbits

$$T.E = E_n = P.E + K.E$$

Kinetic Energy:

$$K.E_n = \frac{1}{2} m v_n^2 = \frac{1}{2} m \left[\frac{Ze^2}{2nh\epsilon_0}\right]^2$$

$$\Rightarrow K.E_n = \frac{1}{2} m \frac{Z^2 e^4}{4n^2 h^2 \epsilon_0^2}$$

$$\Rightarrow K.E_n = \frac{Z^2 e^4 m}{8n^2 h^2 \epsilon_0^2} \longrightarrow (6)$$

$$K.E \propto Z^2$$

$$K.E \propto \frac{1}{n^2}$$

Potential Energy:

$$P.E_n = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_n}$$

$$\Rightarrow P.E_n = \frac{1}{4\pi\epsilon_0}\frac{(Ze)(-e)}{r_n}$$

$$\Rightarrow P.E = \frac{-Ze^2}{4\pi\epsilon_0 \frac{n^2h^2\epsilon_0}{Ze^2\pi m}}$$

$$\Rightarrow P.E = \frac{-Ze^2(Ze^2m)}{4n^2h^2\epsilon_0^2}$$

$$\therefore P.E = \frac{-Z^2e^4m}{4n^2h^2\epsilon_0^2} \rightarrow (7)$$

Total Energy of the electron:

$$\Rightarrow K.E_n = \frac{Z^2 e^4 m}{8 n^2 h^2 \epsilon_0^2} \longrightarrow (6)$$

$$\therefore P.E = \frac{-Z^2 e^4 m}{4 n^2 h^2 \epsilon_0^2} \longrightarrow (7)$$

$$T.E = E_n = P.E + K.E$$

$$\Rightarrow E_n = \frac{-Z^2 e^4 m}{4 n^2 h^2 \epsilon_0^2} + \frac{Z^2 e^4 m}{8 n^2 h^2 \epsilon_0^2}$$

$$\Rightarrow E_n = \frac{-2 Z^2 e^4 m}{8 n^2 h^2 \epsilon_0^2} + \frac{Z^2 e^4 m}{8 n^2 h^2 \epsilon_0^2}$$

$$\therefore E_n = \frac{-Z^2 e^4 m}{8 n^2 h^2 \epsilon_0^2} \longrightarrow (8)$$

$$\boxed{E_n = -K.E = \frac{1}{2} P.E}$$

$$\therefore E_n = \frac{-Z^2 e^4 m}{8 n^2 h^2 \epsilon_0^2} \longrightarrow (8)$$

$$E_n \propto Z^2 \ , \quad E_n \propto \frac{1}{n^2} \Rightarrow \boxed{\frac{E_1}{E_2} = \frac{n_2^2}{n_1^2}}$$

For hydrogen atom, $Z = 1$

$$\therefore E_n = \frac{-me^4}{8 n^2 h^2 \epsilon_0^2}$$

$$\Rightarrow \boxed{E_n = \frac{-13.6}{n^2} \ eV} \longrightarrow (9)$$

BOHR THEORY

DRAWBACKS

Limitations of Bohr Theory

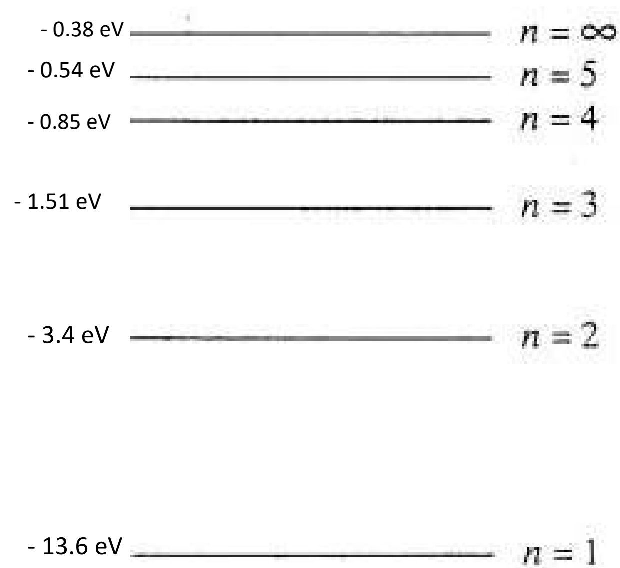
- Spectra of atoms more complex than hydrogen
- Distribution and arrangement of electrons
- Variation in the intensity of the spectral lines
- Fine structure of spectral lines
- Chemical bonding
- Zeeman Effect
- Stark effect

Energy Levels in Hydrogen Atom

Energy of the electron in nth orbit

$$E_n = \frac{-me^4}{8n^2h^2\epsilon_0^2}$$

$$\boxed{E_n = \frac{-13.6}{n^2}\ eV}$$



E_1 = - 13.6 eV

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}$$

$$E_3 = \frac{-13.6}{3^2} = -1.51 \text{ eV}$$

$$E_4 = \frac{-13.6}{4^2} = -0.85 \text{ eV}$$

$$E_5 = \frac{-13.6}{5^2} = -0.54 \text{ eV}$$

$$E_6 = \frac{-13.6}{6^2} = -0.38 \text{ eV}$$

$E_2 - E_1$ = 10.2 eV

$E_3 - E_2$ = 1.89 eV

$E_4 - E_3$ = 0.66 eV

$E_5 - E_4$ = 0.31 eV

$E_6 - E_5$ = 0.16 eV

Wave length or Wave number:

Energy of electron in the nth orbit $E_n = \frac{-z^2 e^4 m}{8 n^2 h^2 \epsilon_0^2} = \frac{-z^2 e^4 m}{8 \epsilon_0^2 h^2}\left(\frac{1}{n^2}\right)$

$$E_{n_1} = \frac{-z^2 e^4 m}{8 \epsilon_0^2 h^2}\left(\frac{1}{n_1^2}\right)$$

$$E_{n_2} = \frac{-z^2 e^4 m}{8 \epsilon_0^2 h^2}\left(\frac{1}{n_2^2}\right)$$

$$\therefore \Delta E = E_2 - E_1$$

$$\Rightarrow \Delta E = \frac{-z^2 e^4 m}{8 \epsilon_0^2 h^2}\left(\frac{1}{n_2^2}\right) - \frac{-z^2 e^4 m}{8 \epsilon_0^2 h^2}\left(\frac{1}{n_1^2}\right)$$

$$h\nu = \frac{z^2 e^4 m}{8 \epsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\nu = \frac{z^2 e^4 m}{8 \epsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$c = \nu\lambda \Rightarrow \nu = \frac{c}{\lambda}$$

$$\therefore \frac{c}{\lambda} = \frac{z^2 e^4 m}{8 \epsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\Rightarrow \frac{1}{\lambda} = \frac{z^2 e^4 m}{8 \epsilon_0^2 h^3 c}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{me^4}{8 \epsilon_0^2 c h^3} = R$$, Rydberg constant

$$= 1.097 \times 10^7 \, m^{-1}$$

$$\therefore \frac{1}{\lambda} = z^2 R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For hydrogen atom, $z = 1$

$$\therefore \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Hydrogen Spectrum:

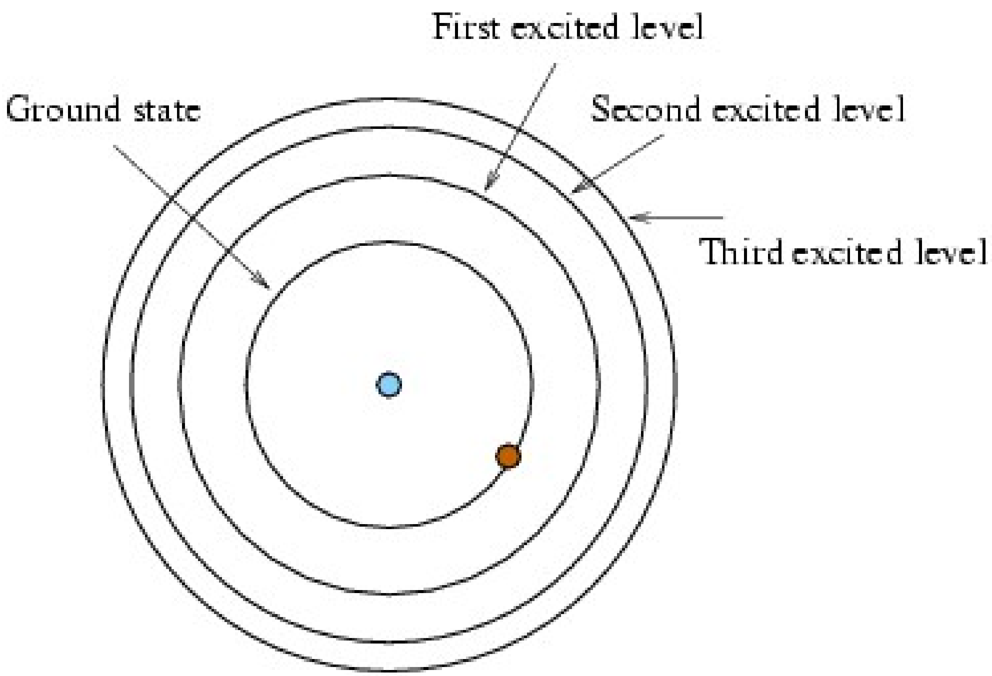


Image source: http://astronomy.nmsu.edu/

Hydrogen Spectral Series

- Lyman series $$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \text{ where } n = 2, 3, 4, \ldots$$
- Balmer series $$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \text{ where } n = 3, 4, 5, \ldots$$
- Paschen series $$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \text{ where } n = 4, 5, 6, \ldots$$
- Bracket series $$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \text{ where } n = 5, 6, 7, \ldots$$
- Pfund series $$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), \text{ where } n = 6, 7, 8, \ldots$$

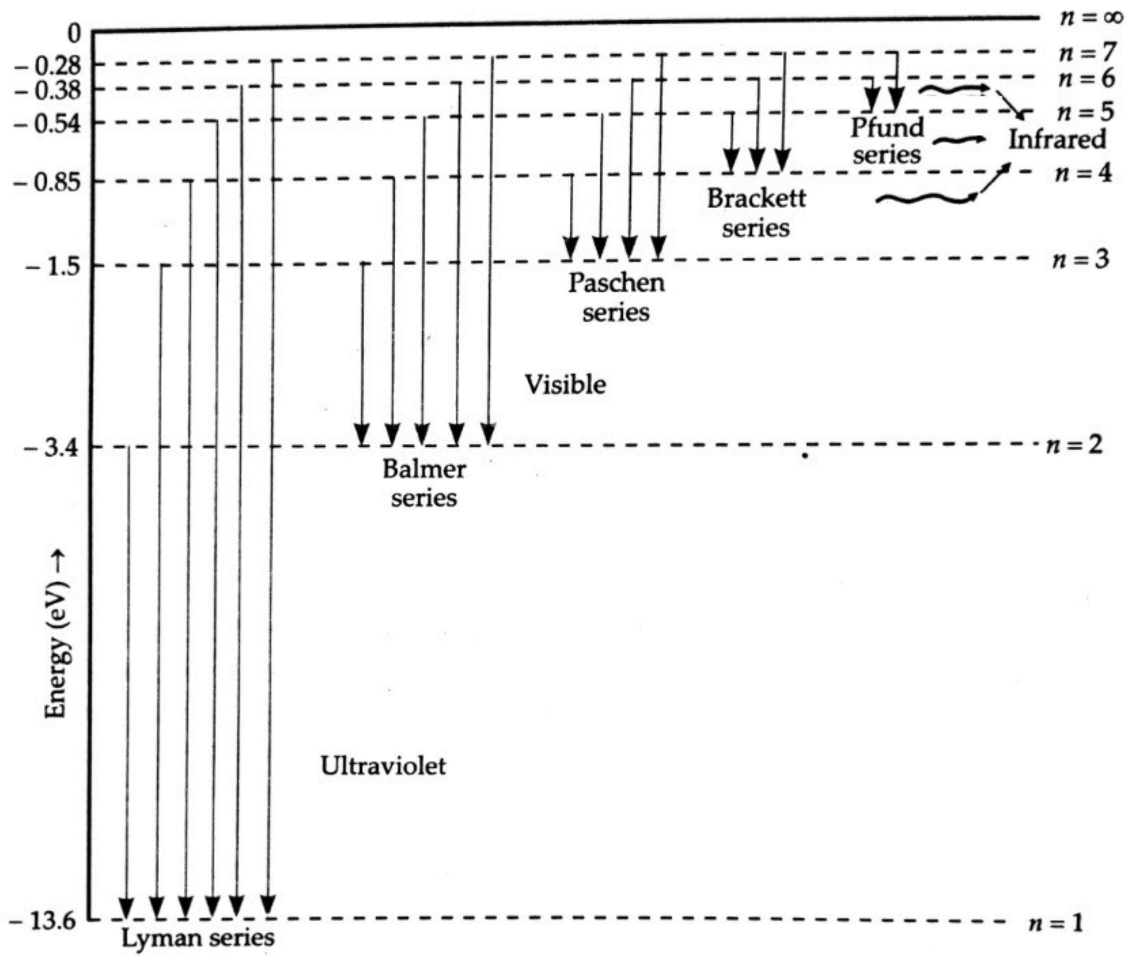


Image source: S L Arora – A good resource book

Hydrogen Spectral Series – Limiting lines

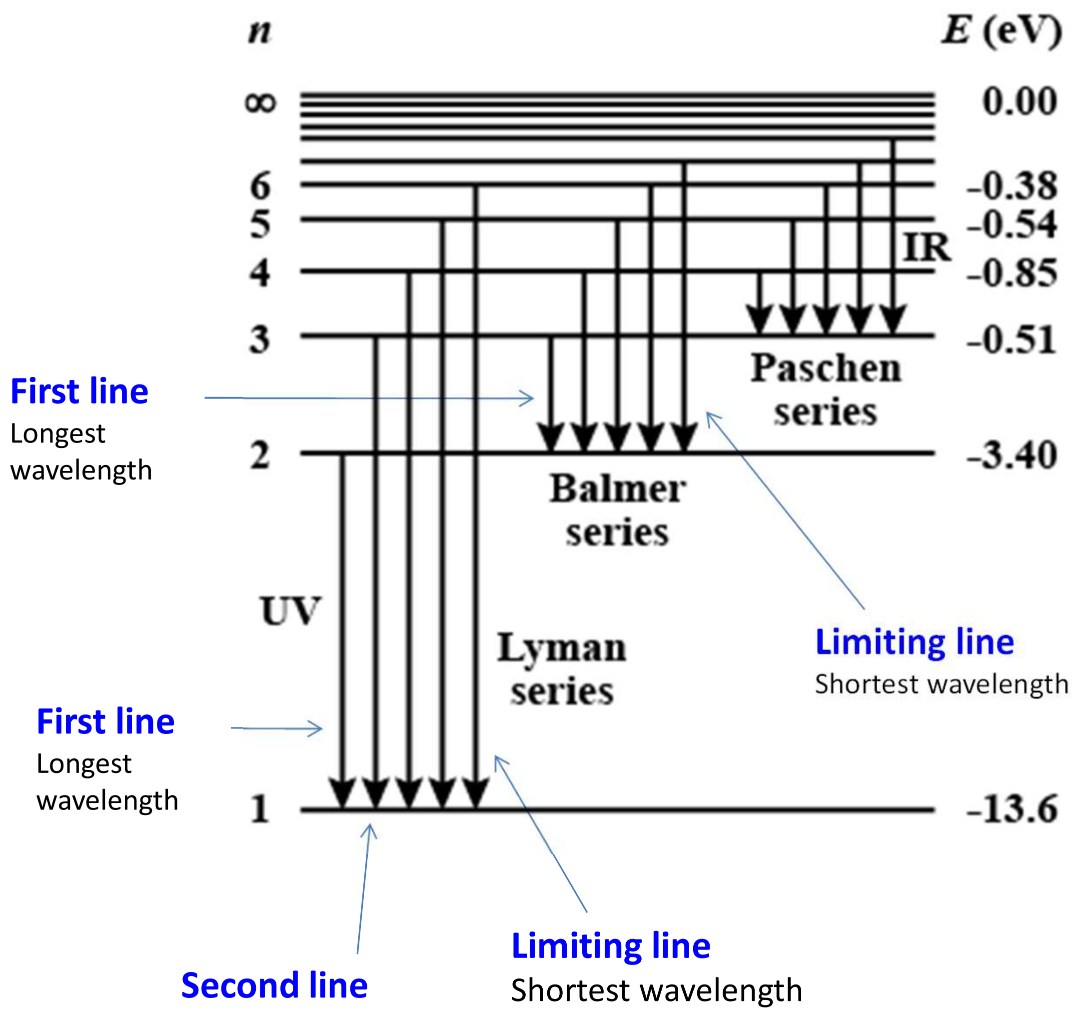


Image source: toppr.com

www.physicspower.com

Lyman series:

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

First line:

For first line $n_1 = 1$ and $n_2 = 2$

$$\Rightarrow \frac{1}{\lambda_1} = R\left[1 - \frac{1}{4}\right] = R\left[\frac{4-1}{4}\right]$$

$$\Rightarrow \frac{1}{\lambda_1} = \frac{3R}{4}$$

$$\therefore \lambda_1 = \frac{4}{3R}$$

$$\Rightarrow \lambda_1 = \frac{4}{3 \times 10.97 \times 10^6} = 1216 A^\circ$$

Second line:

For second line $n_1 = 1$ & $n_2 = 3$

$$\therefore \frac{1}{\lambda_2} = R\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$\Rightarrow \frac{1}{\lambda_2} = R\left[1 - \frac{1}{9}\right] = R\left[\frac{9-1}{9}\right]$$

$$\Rightarrow \frac{1}{\lambda_2} = \frac{8R}{9}$$

$$\Rightarrow \lambda_2 = \frac{9}{8R}$$

$$\Rightarrow \lambda_2 = \frac{9}{8 \times 10.97 \times 10^6} = 1026 \, A^\circ$$

Limiting line:

For Limiting line. $n_1 = 1$ & $n_2 = \infty$

shortest wavelength

$$\therefore \frac{1}{\lambda_\infty} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = R$$

$$\Rightarrow \lambda_\infty = \frac{1}{R} = \frac{1}{10.97 \times 10^6}$$

$$= 912 \, A^\circ$$

Balmer series:

First line:

For first line $n_1 = 2$ & $n_2 = 3$

$$\therefore [\lambda_1]^{-1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\Rightarrow [\lambda_1]^{-1} = R\left[\frac{1}{4} - \frac{1}{9}\right] = R\frac{[9-4]}{36}$$

$$\Rightarrow \lambda_1^{-1} = \frac{5R}{36}$$

$$\Rightarrow \lambda_1 = \frac{36}{5R}$$

$$\lambda_1 = \frac{36}{5 \times 1.97 \times 10^7}$$

$$\Rightarrow \lambda_1 = 6563\,\mathring{A}$$

Limiting line:

For limiting line $n_1 = 2$ & $n_2 = \infty$

$$\therefore \frac{1}{\lambda_\infty} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right]$$

$$\Rightarrow \frac{1}{\lambda_\infty} = R\left[\frac{1}{4}\right]$$

$$\Rightarrow \lambda_\infty = \frac{4}{R}$$

$$\Rightarrow \lambda_\infty = \frac{4}{1.97 \times 10^7} = 3637\,\mathring{A}$$

Paschen Series

First line:

First line wave length (Longest wave length)= 18752 A°

Limiting line:

Limiting line wave length (Shortest wave length) = 8204 A°

PROBLEMS

ATOMS LESSON

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Find the radius of the electron in the 2nd orbit of hydrogen atom

Given

$n = 2$

$r_2 = ?$

$r_n = 0.529\, n^2 A$

$\therefore r_{n=2} = 0.529\,(2)^2$

$r_2 = 0.529 \times 4$

$r_2 = 2.116\, A^\circ$

Calculate the velocity of the electron revolving in the 1st orbit.

Given

$n = 1$

$$v_n = \frac{e^2}{2nh\epsilon_0}$$

$$\therefore v_1 = \frac{e^2}{2h\epsilon_0} = \frac{1.6\times10^{-19}\times1.6\times10^{-19}}{2\times6.625\times10^{-34}\times8.9\times10^{-12}}$$

$$= \frac{2.56\times10^{-38}}{117.925\times10^{-46}}$$

$$= 0.0217\times10^{8}$$

$$\doteq 2.17\times10^{6}\ m/s$$

A photon of energy 12.09 eV is absorbed by an electron in the first energy state of hydrogen atom. Find, to which energy electron will go.

Given $E = h\nu = 12.09\,eV$

$n_i = 1 \quad , \quad n_f = ?$

$E_1 = -13.6\,eV$

$\therefore \quad E_{nf} = -13.6 + 12.09 = -1.51\,eV$

$E_n \propto \frac{1}{n^2} \qquad \frac{E_1}{E_{nf}} = \frac{n_f^2}{1^2} \qquad \Rightarrow n_f^2 = \frac{-13.6}{-1.51} = 9$

$\therefore n_f = 3$

Find the ratio of time periods of the electrons revolving in the first and third energy states of hydrogen atom.

Given

$n_1 = 1 \quad , \quad n_2 = 3$

$\frac{T_1}{T_2} = ?$

$T \propto n^3 \Rightarrow \frac{T_1}{T_2} = \frac{1^3}{3^3} = \frac{1}{27}$

$\therefore T_1 : T_2 = 1 : 27$

Find the ratio of limiting lines wavelength in Lyman, Balmer and Paschen series in hydrogen spectra.

$$\lambda_{L_\infty} : \lambda_{B_\infty} : \lambda_{P_\infty} = ?$$

Limiting line in Lyman series $$\frac{1}{\lambda_{L_\infty}} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = R$$

$$\Rightarrow \lambda_{L_\infty} = \frac{1}{R}$$

Limiting line in Balmer series $$\frac{1}{\lambda_{B_\infty}} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{R}{4}$$

$$\Rightarrow \lambda_{B_\infty} = \frac{4}{R}$$

Limiting line in Paschen series

$$\frac{1}{\lambda_{P_\infty}} = R\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right] = \frac{R}{9}$$

$$\Rightarrow \lambda_{P_\infty} = \frac{9}{R}$$

$$\therefore \lambda_{L_\infty} : \lambda_{B_\infty} : \lambda_{P_\infty} = \frac{1}{R} : \frac{4}{R} : \frac{9}{R}$$

$$= 1 : 4 : 9$$

DE-BROGLIE COMMENT ON BOHR THEORY

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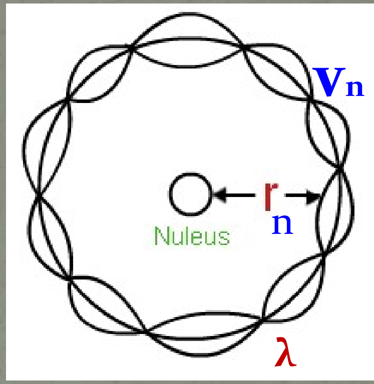
de-Broglie comment on Bohr Theory

Circumference of the orbit $= 2\pi r_n$

For a possible orbit, $2\pi r_n = n\lambda$



According to de-Broglie wavelength, $\lambda = \frac{h}{mv_n}$

v_n - is speed of electron in n^{th} orbit.

$$\therefore \quad 2\pi r_n = n \cdot \frac{h}{mv_n}$$

$$\Rightarrow \boxed{mv_n r_n = n\frac{h}{2\pi}}$$

Angular momentum of the electron revolving around the nucleus quantised