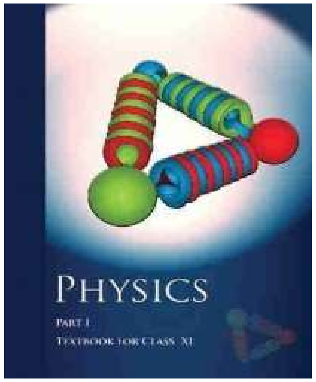

WORK, ENEGY AND POWER

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CHAPTER - 6

WORK, ENERGY AND POWER

Introduction

Notions of work and kinetic energy : The work- energy theorem

Work

Kinetic energy

Work done by a variable force

The work-energy theorem for a variable force

The concept of potential energy

The conservation of mechanical energy

The potential energy of a spring

Various forms of energy : the law of conservation of energy

Power

Collisions

Nothing is removed from this Chapter

TOPICS EXPLAINED IN THIS VIDEO

- Introduction
- Concept of Work done
- Work – Energy Theorem
- Concept of Kinetic Energy
- Concept of Potential Energy
- **Conservation of Energy
- Potential Energy of a Spring
- ***Collisions
- Power

INTRODUCTION

Introduction:

The chapter is not concerned with the common meaning of the word Work…

The aim of this chapter is not only to study the physical quantities work, energy and power but also to understand and apply two methods of solving problems

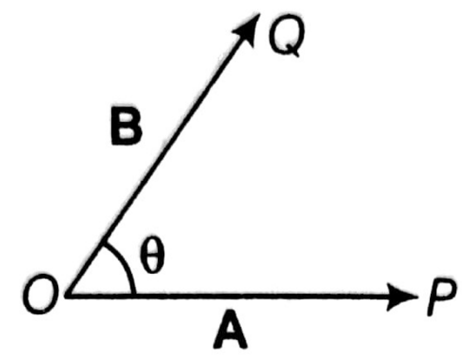
- Work – Energy method
- Law of conservation of energy

SCALAR PRODUCT

Scalar product or Dot product:

It is defined as the product of the magnitudes of vectors **A** and **B** and the cosine angle between them. It is represented by

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$



When the two vectors are parallel, then $\theta = 0°$.

$$\mathbf{A}\cdot\mathbf{B} = AB\cos 0° = AB$$

When the two vectors are mutually perpendicular:

$$\mathbf{A} \cdot \mathbf{B} = AB \cos 90° = 0$$

When the two vectors are antiparallel:

$$\mathbf{A} \cdot \mathbf{B} = AB \cos 180° = -AB$$

Properties of Dot Product:

(*i*) The scalar product is commutative

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

(*ii*) The scalar product is distributive over addition

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

(*iii*) If $\vec{A}$ and $\vec{B}$ are two vectors perpendicular to each other, then their scalar product is zero.

$$\vec{A} \cdot \vec{B} = A\,B \cos 90° = 0$$

(*iv*) If $\vec{A}$ and $\vec{B}$ are two parallel vectors having same direction, then their scalar product has the maximum positive magnitude.

$$\vec{A} \cdot \vec{B} = A\,B \cos 0^\circ = AB$$

(*v*) The scalar product of a vector with itself is equal to the square of its magnitude.

$$\vec{A} \cdot \vec{A} = A \cdot A \cos 0^\circ = A \cdot A = A^2 = |\vec{A}|^2$$

(*vi*) Scalar product of two similar base vectors is unity and that of two different base vectors is zero.

$$\hat{i}\,.\,\hat{i} = (1)(1)\cos 0^\circ = 1 \qquad\qquad \hat{i}\,.\,\hat{i} = \hat{j}\,.\,\hat{j} = \hat{k}\,.\,\hat{k} = 1$$

$$\hat{i}\,.\,\hat{j} = (1)(1)\cos 90^\circ = 0$$

$$\therefore \quad \hat{i}\,.\,\hat{j} = \hat{j}\,.\,\hat{k} = \hat{k}\,.\,\hat{i} = 0$$

Dot product between vectors **A** and **B**:

$$\vec{A}\cdot\vec{B}=\left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\cdot\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$

$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$$

$$\hat{i}\cdot\hat{i}=1 \quad \hat{i}\cdot\hat{j}=0$$

$$\hat{j}\cdot\hat{j}=1 \quad \hat{i}\cdot\hat{k}=0$$

$$\hat{k}\cdot\hat{k}=1 \quad \hat{j}\cdot\hat{k}=0$$

Angle between the vectors:

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}=\frac{A_xB_x+A_yB_y+A_zB_z}{\sqrt{A_x^2+A_y^2+A_z^2}\sqrt{B_x^2+B_y^2+B_z^2}}$$

PROBLEM: *Find the angle between the vectors* $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$ *and* $\vec{B} = -\hat{i} + \hat{j} - 2\hat{k}$.

Solution

$$|\vec{A}| = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}$$

$$|\vec{B}| = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$$

$$\vec{A} \cdot \vec{B} = 1 \times (-1) + 2 \times 1 + (-1) \times (-2)$$

$$= -1 + 2 + 2 = 3$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$$

$$= \frac{3}{\sqrt{6} \times \sqrt{6}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

Hence $\theta = 60^\circ$

WORK

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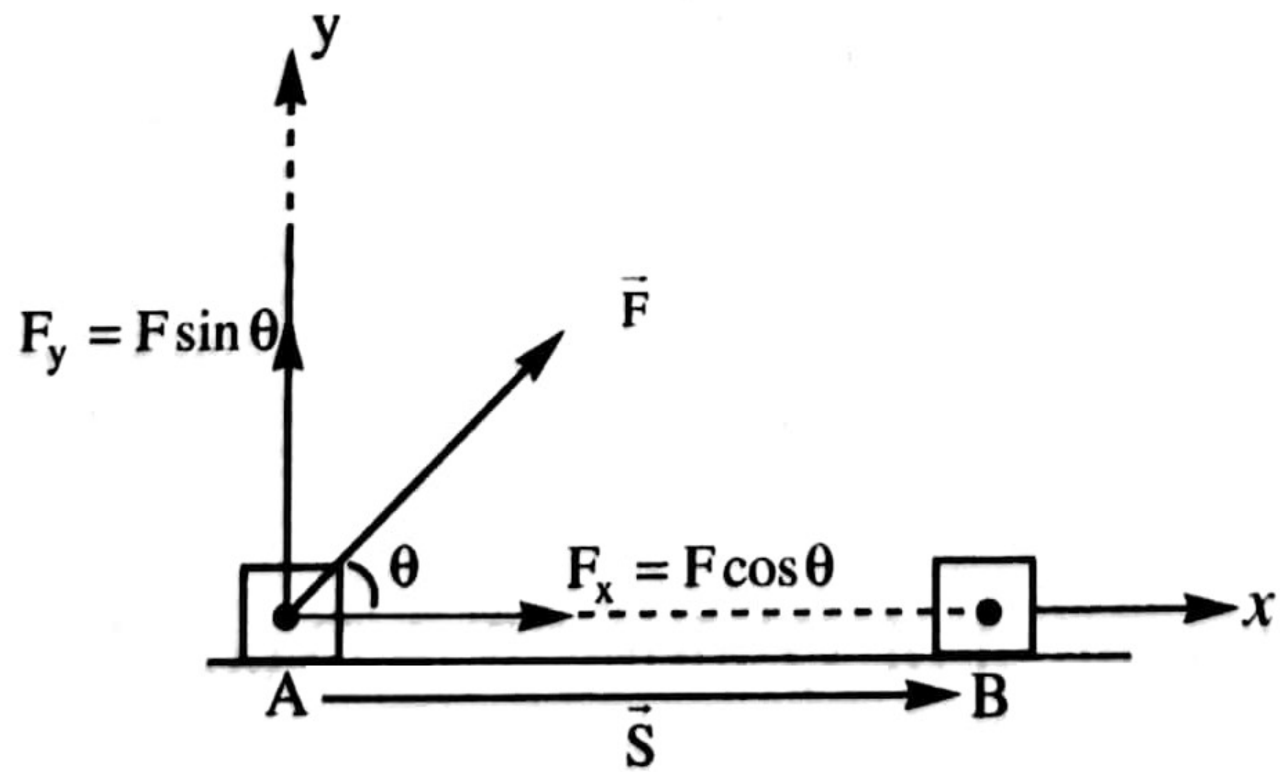
Work done:

Work is said to be done by a force when a body undergoes displacement parallel to the line of action of the force or parallel to one of the components of force applied.



Work done $W = (F\cos\theta)S = FS\cos\theta = \vec{F}.\vec{S}$

Work is a scalar

joule : Work done is said to be one 'joule' if a force of 1 newton displaces a body through a distance of 1 m along the direction of force.

$$1\ J = 1\ N \times 1\ m$$

In CGS system, the unit of work is **erg**

erg : Work done is said to be one 'erg' if a force of one dyne displaces the body through a distance of 1 cm along the direction of force.

$$1 \text{ erg} = 1 \text{ dyne} \times 1 \text{ cm}$$

Relation between joule and erg :

$$1 \text{ J} = 1 \text{N} \times \text{m} = 10^5 \text{ dyne} \times 100 \text{ cm} = 10^7 \text{ dyne cm}$$

$$\boxed{1J = 10^7 erg}$$

Nature of work :

Work done by a force may be positive, negative or zero.

a) **Positive work :** Positive work means that force (or one of its components) is parallel to displacement. If 'θ' is the angle between force and displacement, then $0^0 \leq \theta < 90^0$.

Examples :

(i) When a person lifts a body from the ground, the work done by the lifting force (upward) is positive.

(ii) When a spring is stretched, work done by the external (stretching) force is positive.

b) Negative work :

Negative work means that force (or one of its components) is opposite to displacement. If 'θ' is the angle between force and displacement, then $90^0 < \theta \leq 180^0$.

Examples :

(i) When a person lifts a body from the ground, the work done by the force of gravity (downward) is negative.

(ii) When a body is made to slide over a rough surface, the work done by the frictional force is negative.

c) **Zero Work :**

Under three conditions, work done becomes zero.

If the force is perpendicular to the displacement

$(\theta = 90^0)$

Examples :

When a body moves in a circle the work done by the centripetal force is always zero.

If there is no displacement [S= 0]

Examples :

When a person tries to displace a wall by applying a force and if it does not move, then work done by him is zero.

If the resultant force acting on the body becomes zero. (F = 0)

Examples :

The work done by all forces acting on a raindrop falling down with terminal velocity is zero.

KINETIC ENERGY

Mechanical energy is of two types, namely

1) Kinetic Energy 2) Potential Energy

Kinetic energy is the energy possessed by a body by virtue of its motion.

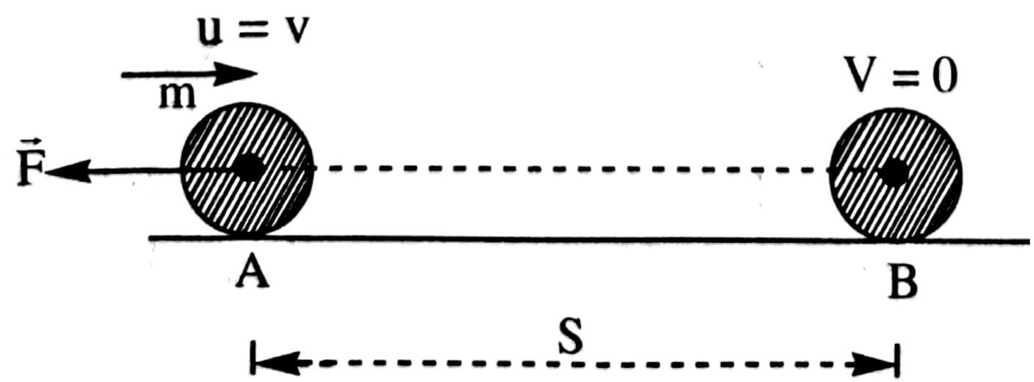
Potential energy is the energy possessed by a body by virtue of its position.

Examples for bodies having Kinetic Energy :

1) A vehicle in motion.
2) Water flowing along a river.
3) A bullet fired from a gun.
4) A satellite going around the earth.

Expression for kinetic energy :

$$v^2 - u^2 = 2as$$



Here initial velocity = v, final velocity

$$0 - v^2 = 2as$$

$$a = \frac{-v^2}{2s}$$

$$F = ma = \frac{-mv^2}{2s}$$

Form Newton's third law of motion, force applied by the body =

$$-F = +\frac{mv^2}{2s}$$

Hence work done by the body against the opposing force acting on it is

$$W = \left(\frac{mv^2}{2s}\right)(s)\cos 0^0 \quad = \frac{1}{2}mv^2$$

$$\therefore \text{ Kinetic energy of the body } \; K = \frac{1}{2}mv^2$$

Relation between Kinetic energy and Linear momentum:

$$K.E = \frac{1}{2}mv^2 \quad = \frac{m^2v^2}{2m}$$

$$= \frac{p^2}{2m} \quad [\because p = mv]$$

WORK – ENERGY THEOREM

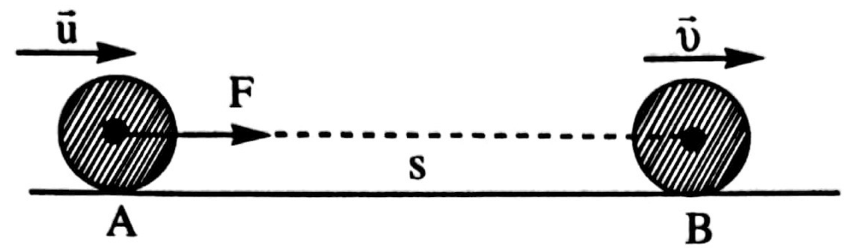
Work – Energy Theorem:

The work done on a particle by the net force is equal to the change in its kinetic energy

Proof:

$$W = FS = maS$$

$$a = \frac{v^2 - u^2}{2S}$$



$$= m\left(\frac{v^2 - u^2}{2S}\right)S$$

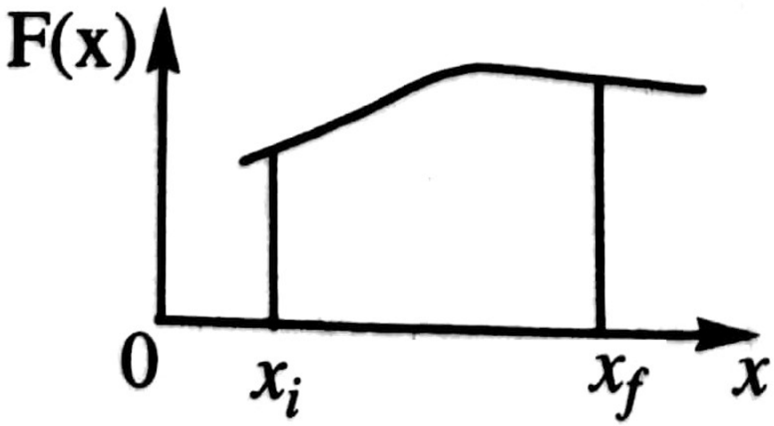
$$= m\left(\frac{v^2 - u^2}{2}\right) \quad = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

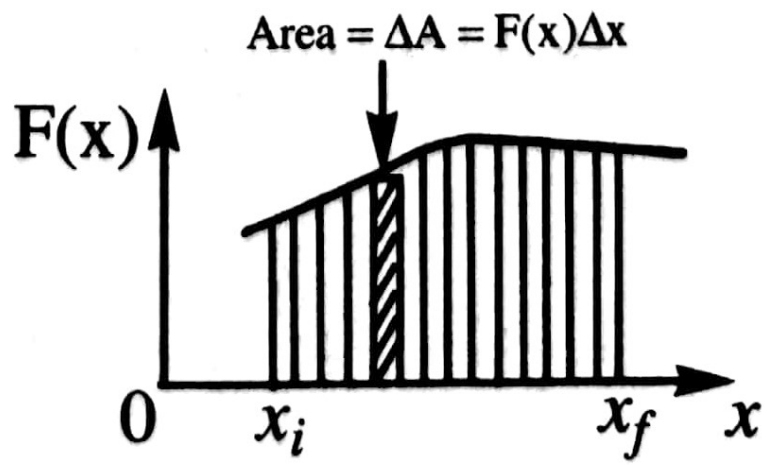
$$W = K_f - K_i$$

Hence Work – Energy theorem is proved.

WORK DONE BY VARIABLE FORCE

Work done by variable force:





$$\Delta W = F(x).\Delta x$$

$\therefore$ Total work done by variable force is sum all the works ΔW i.e, $W = \sum_{x_i}^{x_f} F(x)\Delta x$

$$W = \int_{x_i}^{x_f} F(x)dx$$

WORK-ENERGY THEOREM FOR VARIABLE FORCE

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Work – Energy theorem for variable force:

let F is the variable for applied on a particle

Let the particle displaced from x_i to x_f

and kinetic energy changed from K_i to K_f

ΔK = Work done

WKT $\quad K = \frac{1}{2} m v^2$

$$\frac{dk}{dt} = \frac{1}{\cancel{2}} m \cdot \cancel{2} v \frac{dv}{dt}$$

$$\Rightarrow \frac{dk}{dt} = m \frac{dv}{dt} \cdot \frac{dx}{dt}$$

$$\Rightarrow \frac{dk}{dt} = ma \frac{dx}{dt}$$

$$\Rightarrow \frac{dk}{dt} = F \cdot \frac{dx}{dt}$$

$$\Rightarrow dk = F \cdot dx$$

$$\int_{k_i}^{k_f} dk = \int_{x_i}^{x_f} F \cdot dx$$

$$\left[K\right]_{K_i}^{K_f} = W$$

$$\Rightarrow K_f - K_i = W$$

$$\boxed{\therefore \Delta K = W}$$

POTENTIAL ENERGY

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Potential Energy:

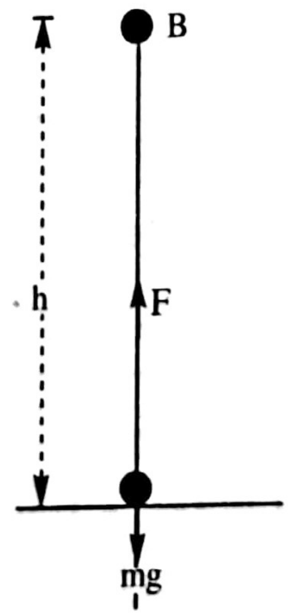
Potential energy of a body is the energy possessed by a body by virtue of its position or configuration

$$W = Fs$$

$$(\because F = mg; s = h)$$

$$W = mgh.$$

This work gets stored as potential energy of the body. Potential energy of the body $\boxed{U = mgh.}$



Potential Energy of a spring:

The potential energy of a spring

$$dW = F dx$$

$$= -kx\,dx$$

$\therefore$ Work done to stretch the spring

$$W = \int dW = \int_0^x -kx\,dx$$

$$\Rightarrow \quad W = -k\left[\frac{x^2}{2}\right]_0^x$$

$$\therefore \quad W = -\frac{1}{2}kx^2$$

$$\boxed{\therefore \quad U = \frac{1}{2}kx^2}$$

LAW OF CONSERVATION OF ENERGY

Law of conservation of energy:

The **law of conservation of energy** states that energy can neither be created nor destroyed - only converted from one form of energy to another.

Law of conservation of mechanical energy or Principle of conservation of mechanical energy:

The principle of the **conservation of mechanical energy** states that the total **mechanical energy** in a system (the sum of the potential and kinetic energies) remains constant as long as the only forces acting are conservative forces.

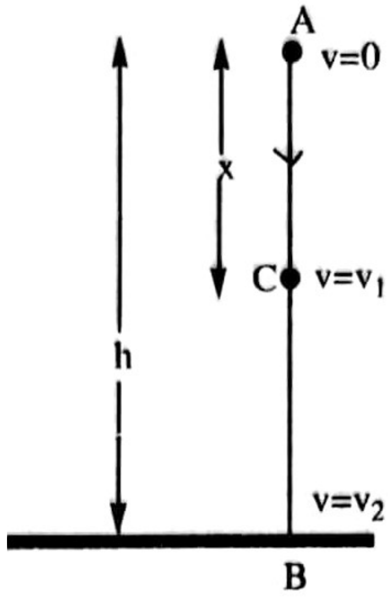
Law of conservation of energy in case of a freely falling body:

At point A :-

P.E of the body $= mgh$

velocity of the body $v = 0$

$\therefore$ K.E of the body $= \frac{1}{2}mv^2 = 0$



$\therefore$ Total energy at A = P.E + K.E.

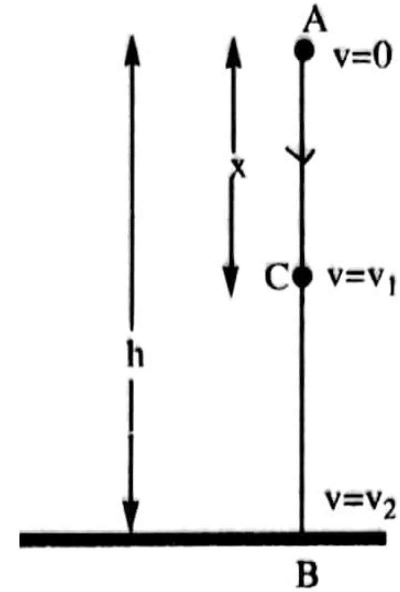
$$= mgh + 0$$

$$= mgh \longrightarrow (1)$$

<u>At point C :-</u>

P.E of the body $= mg[h-x] = mgh - mgx$

$u = 0$, $v = v_1$, $a = g$ & $s = x$



From the formula $v^2 - u^2 = 2as$

$$\Rightarrow v_1^2 - 0^2 = 2gx$$

$$\Rightarrow v_1^2 = 2gx$$

$\therefore$ K.E of the body at $C = \frac{1}{2} m v_1^2$

$$= \frac{1}{2} m (2gx)$$

$$= mgx$$

Total energy at C = P.E + K.E

$$= mgh - mgx + mgx$$

$$= mgh \longrightarrow (2)$$

At point B :-

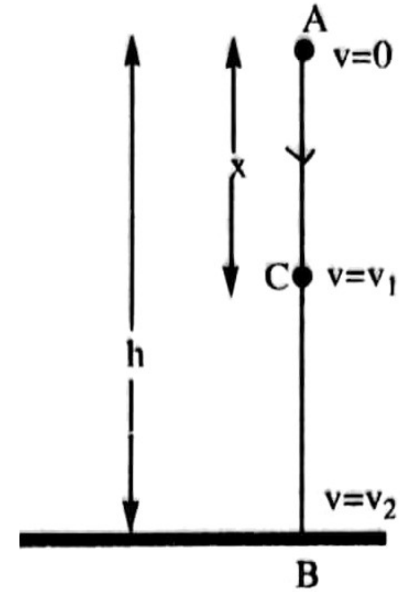
P.E at B $= mg(0) = 0$

$u = 0$, $v = v_2$, $a = g$ & $s = h$

$\therefore \quad v^2 - u^2 = 2as$

$v_2^2 - 0^2 = 2gh$

$\Rightarrow v_2^2 = 2gh$



$\therefore$ K.E at B $= \frac{1}{2}mv^2$

$$= \frac{1}{2}m(2gh)$$

$$= mgh$$

$\therefore$ Total energy at B $=$ P.E $+$ K.E

$$= 0 + mgh$$

$$= mgh \longrightarrow (3)$$

From equations 1, 2 and 3 total energy at points A, B and C are same i.e. total energy remains constant. Hence the law of conservation of energy proved.

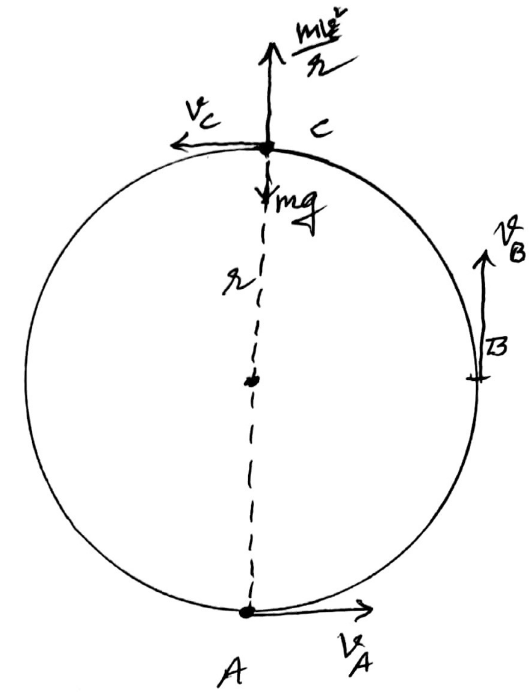
Vertical circle:

At point c :-

$$\frac{mv_c^2}{r} = mg$$

$$\Rightarrow v_c^2 = gr$$

$$\Rightarrow \boxed{v_c = \sqrt{gr}}$$



At point A :-

According to conservation of energy

$$T.E_A = T.E_C$$

$$0 + \frac{1}{2} m v_A^2 = mg(2r) + \frac{1}{2} m v_C^2$$

$$\Rightarrow \frac{1}{2} m v_A^2 = 2mgr + \frac{1}{2} m v_C^2$$

$$\Rightarrow \frac{1}{2} m v_A^2 = 2 \frac{2m}{2} gr + \frac{1}{2} m v_C^2$$

$$\Rightarrow \quad v_A^2 = 4gr + gr \quad [\because v_C^2 = gr]$$

$$\Rightarrow \quad v_A^2 = 5gr$$

$$\boxed{\therefore v_A = \sqrt{5gr}}$$

At point B :-

According to conservation of energy

$$T.E_B = T.E_A$$

$$mgr + \frac{1}{2}mv_B^2 = 0 + \frac{1}{2}mv_A^2$$

$$\Rightarrow \frac{1}{2}mv_B^2 = \frac{1}{2}mv_A^2 - mgr$$

$$\Rightarrow \frac{1}{2}mv_B^2 = \frac{1}{2}mv_A^2 - 2\frac{m}{2}gr$$

$$\Rightarrow v_B^2 = 5gr - 2gr$$

$$\Rightarrow v_B^2 = 3gr$$

$$\boxed{\therefore v_B = \sqrt{3gr}}$$

CONCERVATION
OF
LINEAR MOMENTUM

Law of conservation of linear momentum:

Under the absence of external force, the total linear momentum of an isolated system of particles is conserved i.e. total linear momentum of the system is constant.

Proof:

When one object exerts a force on other object,

the other object also exerts an equal & opposite force on the first object

Force exerted by Object A = Force exerted by Object B

$$m_1 \frac{(v_1 - u_1)}{t} = -m_2 \frac{(v_2 - u_2)}{t}$$

$$m_1 (v_1 - u_1) = -m_2 (v_2 - u_2)$$

$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$$m_1v_1 + m_2v_2 = m_2u_2 + m_1u_1$$

∴ Final Momentum of 2 objects = Original Momentum of 2 objects

Thus, momentum is conserved

COLLISIONS

Collisions:

Collision is short-duration interaction between two bodies or more than two bodies simultaneously causing change in motion of bodies involved due to internal forces acted between them during this.

Or

Redistribution of momentum of the bodies if two or more bodies interacted is called collision.

Types of Collisions:

On the basis of conservation of total kinetic energy of colliding particles, collisions are classified into two types.

a) Elastic collisions and

b) Inelastic collisions

a) Perfectly elastic Collisions:

The collisions, in which both momentum and kinetic energy remain constant or unchanged are known as elastic collisions.

Examples of perfectly elastic collisions are

i) Collision between atomic particles

ii) Collision of α-particle with nucleus

b) Inelastic (or semi elastic) Collisions:

The collisions, in which kinetic energy is not conserved but law of conservation of momentum holds good are known as inelastic collisions.

Examples of inelastic collisions are

i) Collision between two billiard balls

ii) Collision between two automobiles on a road

c) Perfectly Inelastic Collisions:

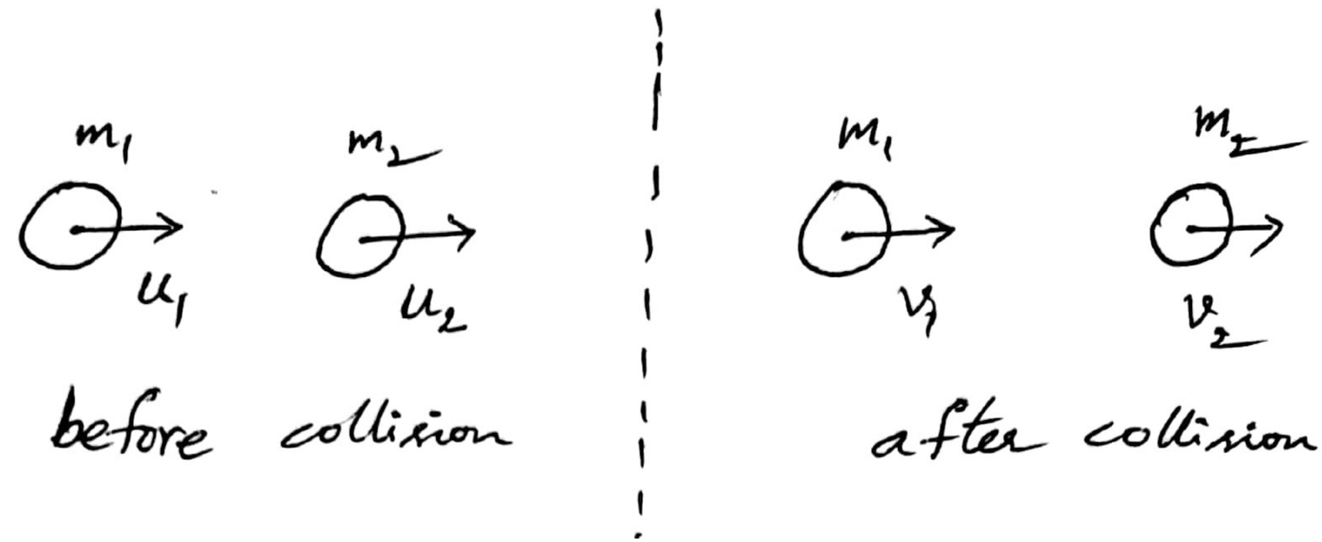
If in a collision, the two colliding bodies stick together and move with a common velocity, then the collision is called perfectly inelastic collision.

Examples of perfectly inelastic collision

Collision between bullet and block of wood into which it is fired, when the bullet remains embedded in the block.

Elastic Collisions in One Dimension

Elastic Collisions in one dimension:



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \longrightarrow (1)$$

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\Rightarrow m_1[u_1^2 - v_1^2] = m_2[v_2^2 - u_2^2] \longrightarrow (2)$$

Eqn (1), we can write as

$$m_1[u_1 - v_1] = m_2[v_2 - u_2] \longrightarrow (3)$$

Dividing eqns (2) by eqn (3), we get

$$\frac{u_1^2 - v_1^2}{(u_1 - v_1)} = \frac{v_2^2 - u_2^2}{(v_2 - u_2)}$$

$$\Rightarrow \frac{(u_1-v_1)(u_1+v_1)}{(u_1-v_1)} = \frac{(v_2-u_2)(v_2+u_2)}{(v_2-u_2)}$$

$$\Rightarrow u_1+v_1 = v_2+u_2 \longrightarrow (4)$$

$$\Rightarrow v_1 = v_2+u_2-u_1$$

$$m_1u_1+m_2u_2 = m_1[v_2+u_2-u_1]+m_2v_2$$

$$m_1u_1+m_2u_2 = \underline{m_1v_2}+m_1u_2-m_1u_1+\underline{m_2v_2}$$

$$\Rightarrow (m_1+m_2)v_2 = m_1u_1+m_2u_2-m_1u_2+m_1u_1$$

$$\Rightarrow (m_1+m_2)v_2 = 2m_1u_1 + (m_2-m_1)u_2$$

$$\therefore v_2 = \frac{2m_1}{(m_1+m_2)}u_1 + \frac{(m_2-m_1)}{(m_1+m_2)}u_2 \longrightarrow (5)$$

From eqn (4) $v_2 = u_1 + v_1 - u_2$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2[u_1+v_1-u_2]$$

$$\Rightarrow m_1u_1 + m_2u_2 = \underline{m_1v_1} + m_2u_1 + \underline{m_2v_1} - m_2u_2$$

$$\Rightarrow (m_1+m_2)v_1 = m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2$$

$$\Rightarrow (m_1+m_2)v_1 = (m_1-m_2)u_1 + 2m_2u_2$$

$$\therefore v_1 = \frac{(m_1-m_2)}{(m_1+m_2)}u_1 + \frac{2m_2}{(m_1+m_2)}u_2 \longrightarrow (6)$$

Special cases

Case (1) If $m_1 = m_2 = m$ then eqn (5) & eqn (6) becomes

$$v_1 = 0 + \frac{2m}{(m+m)}u_2 \Rightarrow v_1 = u_2$$

$$v_2 = \frac{2m}{(m+m)}u_1 + 0 \Rightarrow v_2 = \frac{2m_1}{(m_1+m_2)}u_1 \Rightarrow v_2 = u_1$$

Case ② If $m_1 = m_2 = m$ & $u_2 = 0$

then $v_1 = 0 + 0 = 0$

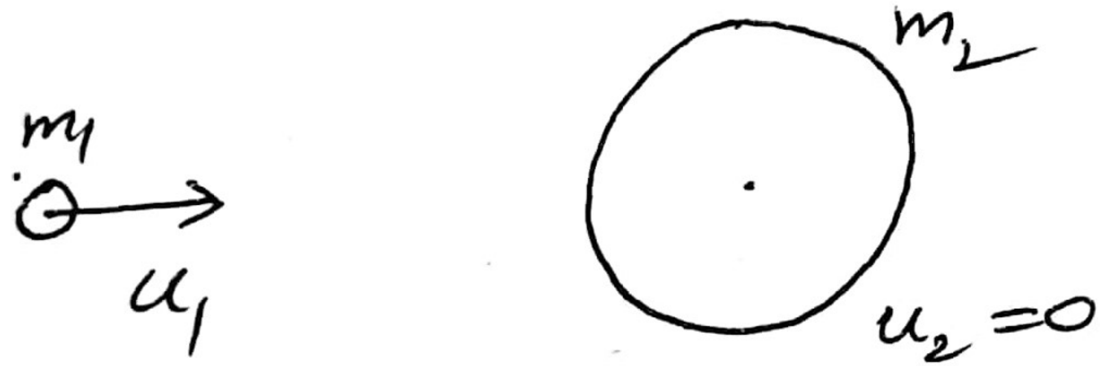
$$v_2 = \frac{2m}{(m+m)} u_1 = u_1$$

Case ③:- If $m_2 >> m_1$ & $u_2 = 0$

then $v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1$

$$= \frac{-m_2}{m_2} u_1 \Rightarrow v_1 = -u_1$$

$$v_2 = \frac{2m_1}{(m_1+m_2)} u_1 \approx 0$$



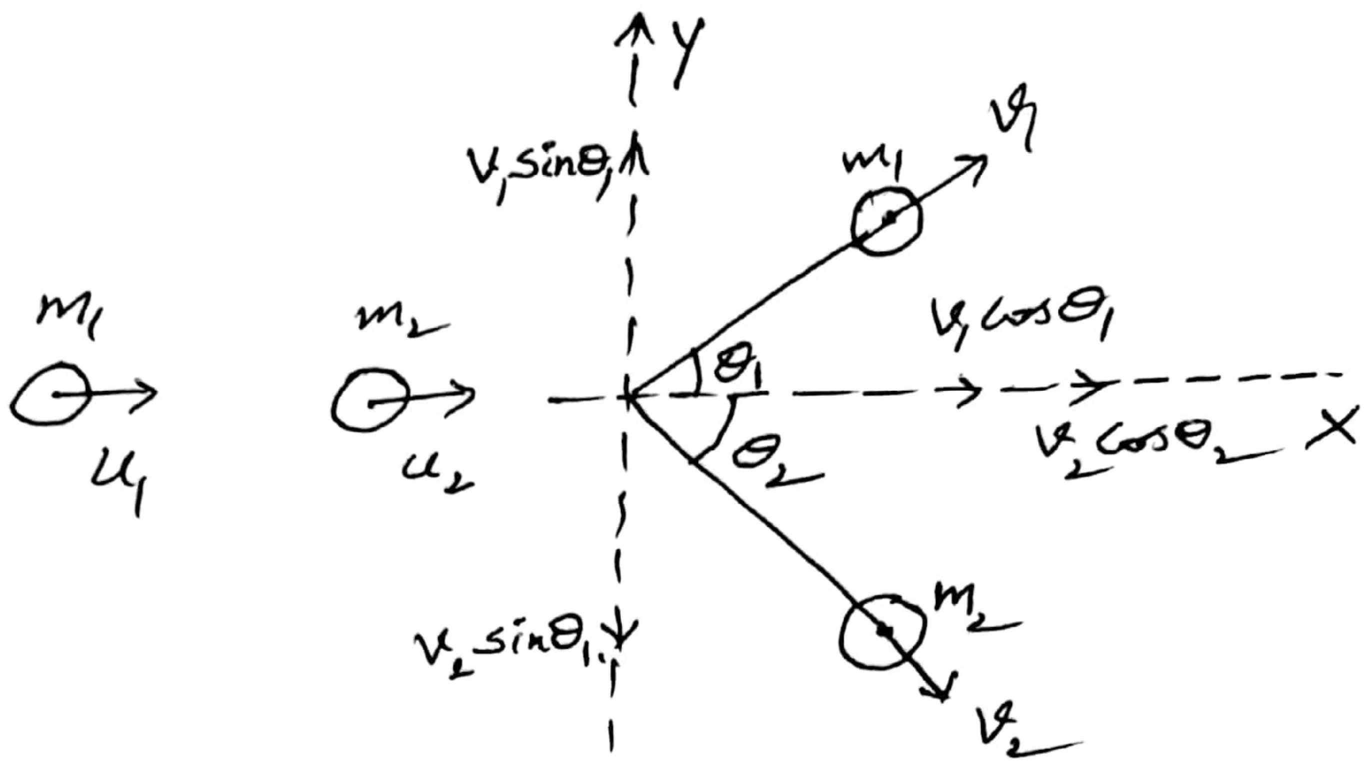
Case (4) :- If $m_1 >> m_2$ & $u_2 = 0$

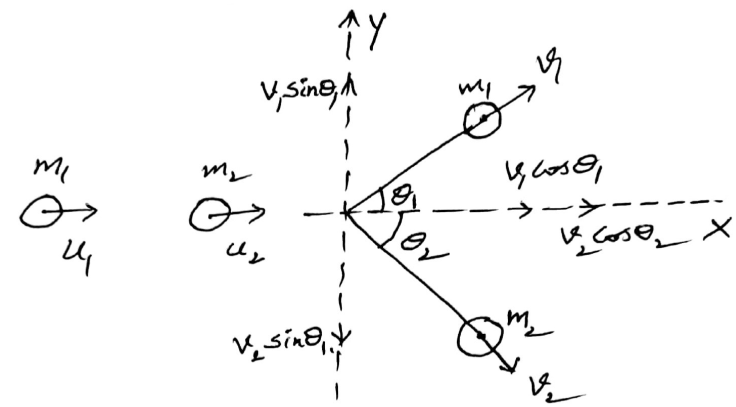
then $$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + 0$$

$$= \frac{m_1}{m_1} u_1 \Rightarrow v_1 = u_1$$

$$v_2 = \frac{2m_1}{(m_1 + m_2)} u_1 + 0 = 2u_1$$

Elastic Collision in two dimension:





Along X-axis

$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \rightarrow (1)$$

Along Y-axis

$$0 + 0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 \rightarrow (2)$$

$$m_2 v_2 \sin\theta_2 = m_1 v_1 \sin\theta_1$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \rightarrow (3)$$

Special Case:

$$m_1 = m_2 = m \quad \& \quad u_2 = 0$$

then eqn (1), (2) & (3) becomes

$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \rightarrow (1)$$

$$u_1 = v_1 \cos\theta_1 + v_2 \cos\theta_2$$

$$\Rightarrow v_2 \cos\theta_2 = u_1 - v_1 \cos\theta_1 \longrightarrow (4)$$

$$0 + 0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 \rightarrow (2)$$

$$v_2 \sin\theta_2 = v_1 \sin\theta_1 \longrightarrow (5)$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \longrightarrow (3)$$

$$u_1^2 = v_1^2 + v_2^2 \longrightarrow (6)$$

Squaring and adding eqns (4) & (5), we get

$$\Rightarrow v_2\cos\theta_2 = u_1 - v_1\cos\theta_1 \longrightarrow (4) \qquad v_2\sin\theta_2 = v_1\sin\theta_1 \longrightarrow (5)$$

$$v_2^2 = (u_1 - v_1\cos\theta_1)^2 + v_1^2\sin^2\theta_1$$

$$\Rightarrow v_2^2 = u_1^2 + \underline{v_1^2 \cos^2\theta_1} - 2u_1 v_1 \cos\theta_1 + \underline{v_1^2 \sin^2\theta_1}$$

$$\Rightarrow v_2^2 = u_1^2 + v_1^2 - 2u_1 v_1 \cos\theta_1$$

$$\Rightarrow v_2^2 = v_1^2 + v_2^2 + v_1^2 - 2u_1 v_1 \cos\theta_1$$

From eqn (6) $u_1^2 = v_1^2 + v_2^2$

$$\Rightarrow 2v_1^2 = 2u_1 v_1 \cos\theta_1$$

$$\Rightarrow \quad \boxed{v_1 = u_1 \cos\theta_1} \longrightarrow (7)$$

$$u_1^2 = v_1^2 + v_2^2 \longrightarrow (6)$$

From eqns (6) & (7)

$$u_1^2 = u_1^2 \cos^2\theta_1 + v_2^2$$

$$\Rightarrow v_2^2 = u_1^2 \left[1 - \cos^2\theta_1\right]$$

$$\Rightarrow v_2^2 = u_1^2 \sin^2\theta_1$$

$$\boxed{\therefore v_2 = u_1 \sin\theta_1} \longrightarrow (8)$$

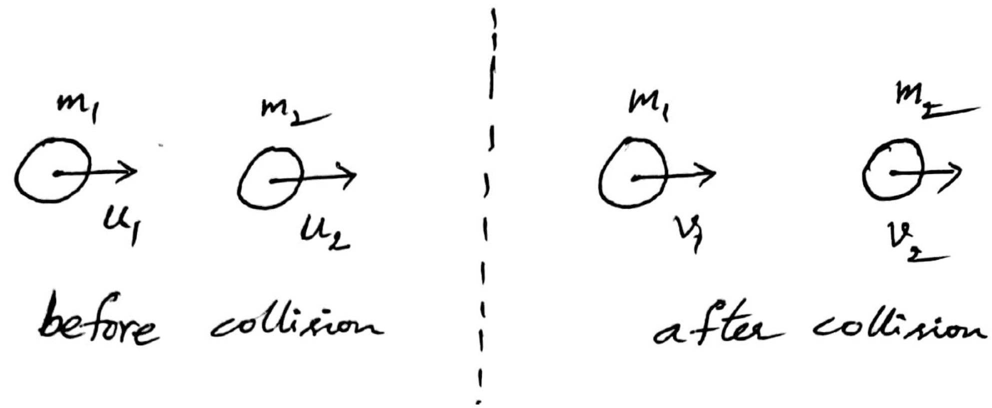
$$\boxed{v_1 = u_1 \cos\theta_1} \longrightarrow (7)$$

$$\boxed{\therefore \theta_1 + \theta_2 = 90^\circ} \longrightarrow (9)$$

Hence, after collision two bodies will move perpendicular to each other.

Coefficient of restitution

Coefficient of restitution:



Then the coefficient of restitution (e) is defined as the ratio of the relative velocity of separation $(v_2 - v_1)$ after the collision to the relative velocity of approach $(u_1 - u_2)$ before collision.

$$e = \frac{\text{relative velocity of separation after collision along the line of impact}}{\text{relative velocity of approach before collision along the line of impact}}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

e has no units and no dimensions

For a perfectly elastic collision, $e = 1$

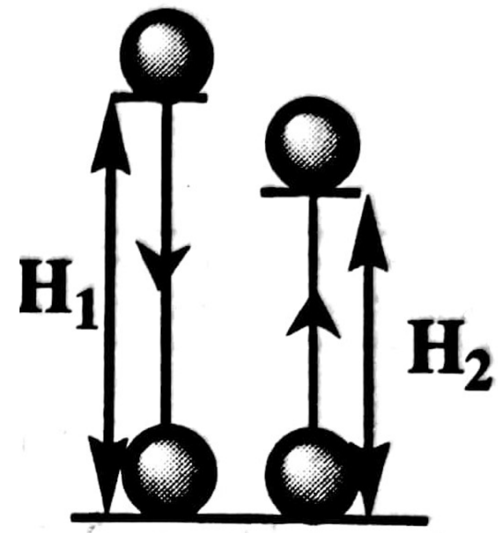
For a perfectly inelastic collision, $e = 0$

For other collisions, e lies between 0 and 1

Coefficient of restitution is termed as degree of elasticity

$$v^2 = 2gS$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{-v_1}{u_1}$$



But $u_1 = \sqrt{2gH_1}$

$$v_1 = -\sqrt{2gH_2}$$

v_1 is in opposite direction to u_1

$$\therefore e = \frac{-v_1}{u_1} = \frac{\sqrt{2gH_2}}{\sqrt{2gH_1}}$$

$$= \sqrt{\frac{H_2}{H_1}}$$

Thus 'e' is a number and has no units and is dimensionless.

e = 1 - perfectly elastic collision

$$u_1 + v_1 = v_2 + u_2 \longrightarrow (4)$$

0 < e < 1 - semi elastic collision

e = 0 - perfectly inelastic collision

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

POWER

Power:

The time rate of doing work is called power.

If W is the total work done by a force in a time interval t then the average power is

$$P_{av} = \frac{W}{t} = \frac{total\ work}{total\ time}$$

Instantaneous power is the dot product of force and velocity of body, provided the force does not change with time.

The instantaneous power is

$$P_{inst} = \vec{F}.\vec{V} = FV\cos\theta$$

$$P_{inst} = \underset{\Delta t \to 0}{Lt} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

$$dW = \vec{F}.\overrightarrow{dS}$$

$$P_{inst} = \vec{F}.\frac{\overrightarrow{dS}}{dt}$$

$$P_{inst} = \vec{F}.\vec{V}$$

Power is a scalar quantity with dimensions $[ML^2T^{-3}]$

S.I unit of power is J/sec (or) watt (W)

CGS unit of power is erg/sec

Practical unit of power is horse power (Hp)

$$1Hp = 746W$$

Watt : The power of an agent is said to be one watt, if one joule of work is done in one second.

$$1 \text{ watt} = \frac{1\, joule}{1\, s} = 1 \text{ J s}^{-1}$$

THANK YOU