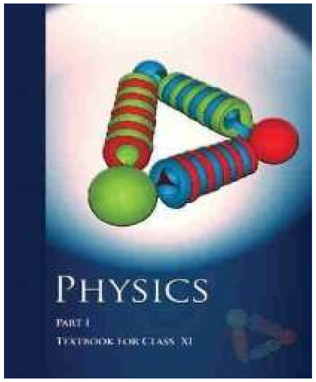


Motion in a Straight Line

CHAPTER - 3

MOTION IN A STRAIGHT LINE

Introduction

Position, path length and displacement

Average velocity and average speed

Instantaneous velocity and speed

Acceleration

Kinematic equations for uniformly accelerated motion

Relative velocity

Removed Topic

Frame of reference

SUB TOPICS :

Introduction to motion in a straight line

Some definitions

Displacement and Distance

Uniform speed, Non-uniform speed & Average speed

Uniform velocity, Non-uniform velocity & Average velocity

Acceleration - Retardation

Graphical Analysis

Derivations: $v = u + at$, $v^2 - u^2 = 2aS$, $S = ut + \frac{1}{2} at^2$...

Freely falling body

Vertically projected body – H_{max}, Time of flight, etc.,.

Introduction to Motion in a Straight Line

TYPES OF MOTION:

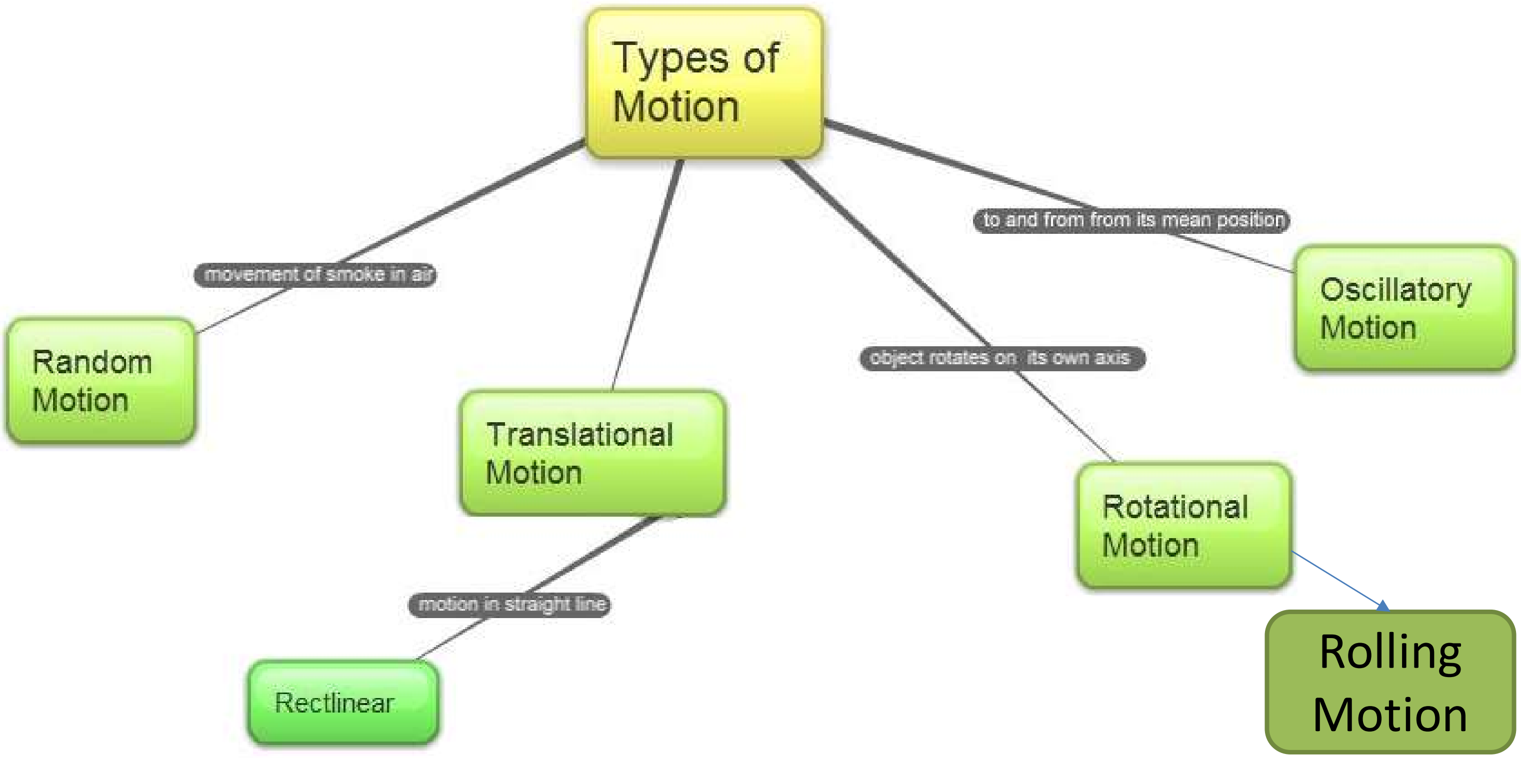
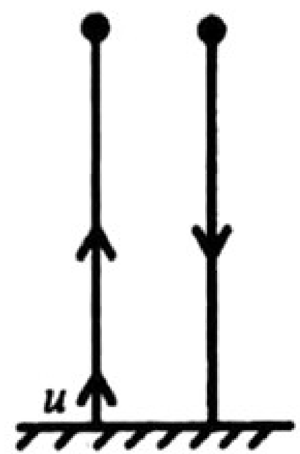


Image source: https://academicseasy.com/

RECTILINEAR MOTION:

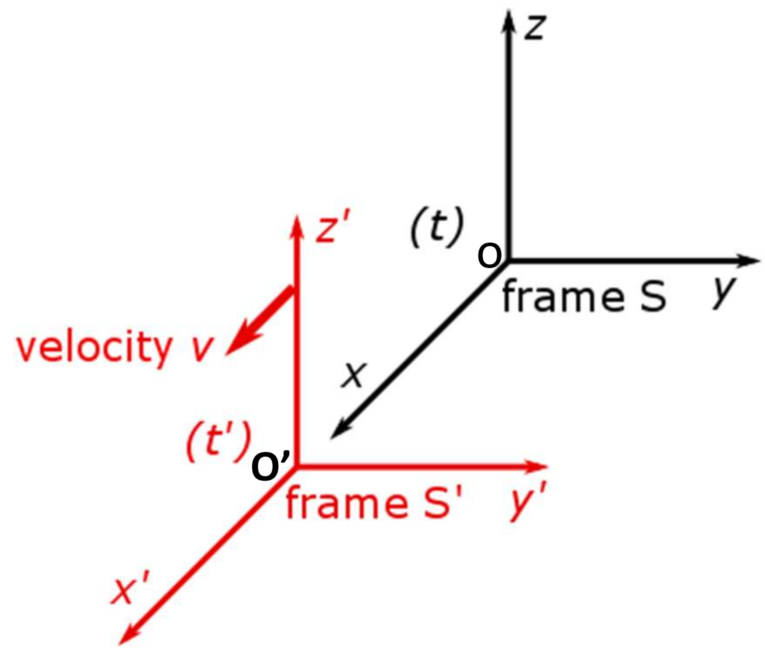
The study of motion of objects along straight line is known as rectilinear motion

Eg: Freely falling body, body thrown vertically upward, etc.,



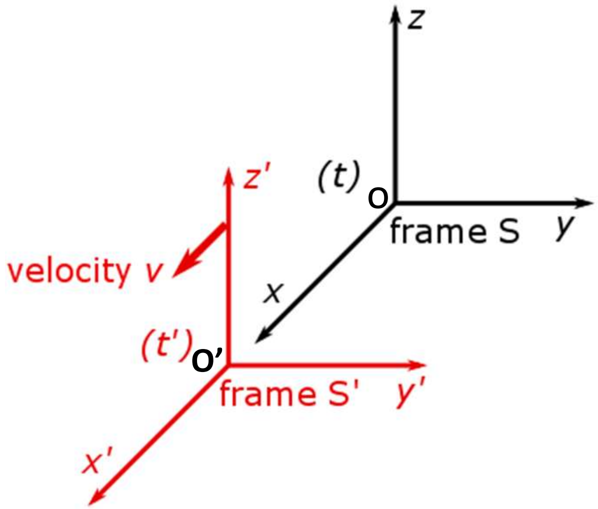
Frame of reference:

The coordinate system along with a clock constitute a frame of reference.



Eg: Laboratory frame of reference, Center of mass frame of reference, etc.,

Rest, Motion:

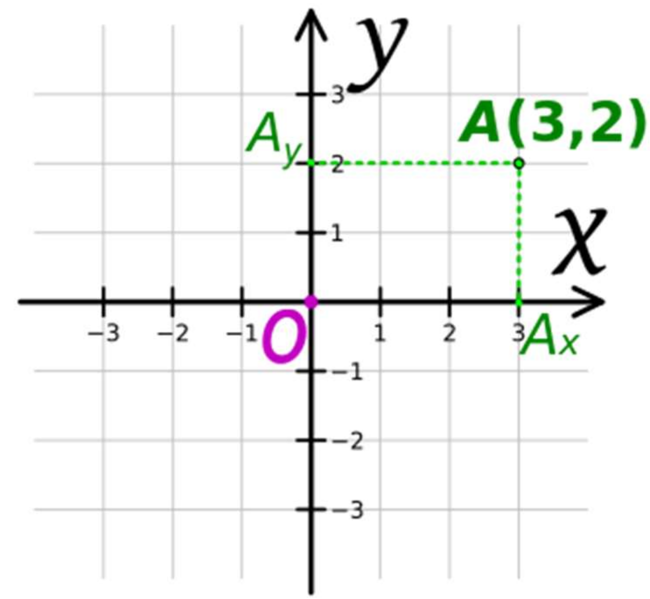


If a particle's position does not change either with respect to time or with respect to a fixed point, then it is said to be at rest.

If a particle's position is continuously changing with respect to time or with respect to a fixed point, then it is said to be in motion.

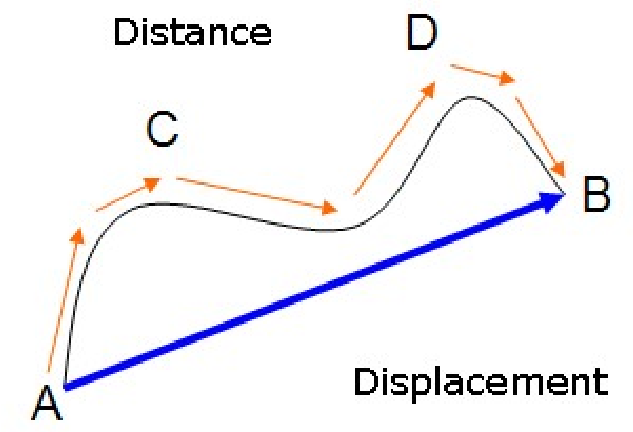
Position, Path length:

The coordinates of a coordinate system describes the position of an object with respect to the origin of the system.



Length of actual path between initial and final positions is called distance or path length.

Displacement, Distance:



The shortest distance directed from the initial position to the final position irrespective of the path is called displacement.

Length of actual path between initial and final positions is called distance or path length.

Unit: In MKS - meter / In CGS - Centimeter.

SPEED

Speed:

The distance travelled by a body in unit time is called it’s speed.

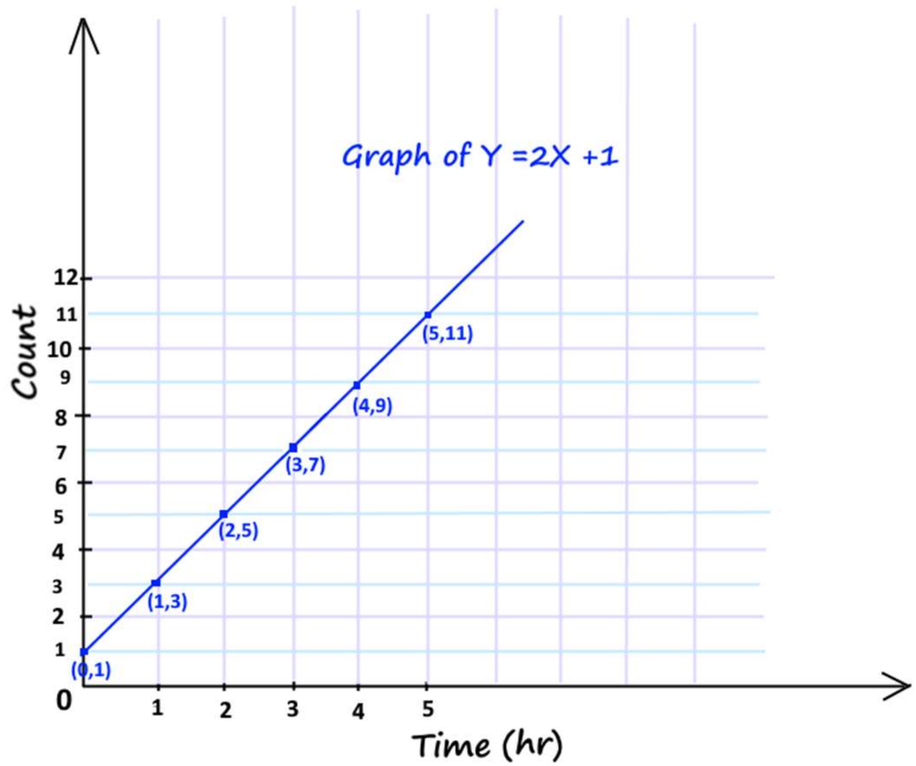
Uniform Speed:

If a particle moving along a straight line travels equal distances in equal intervals of time, then the particle is moving with uniform speed.

Non-uniform Speed or Variable Speed:

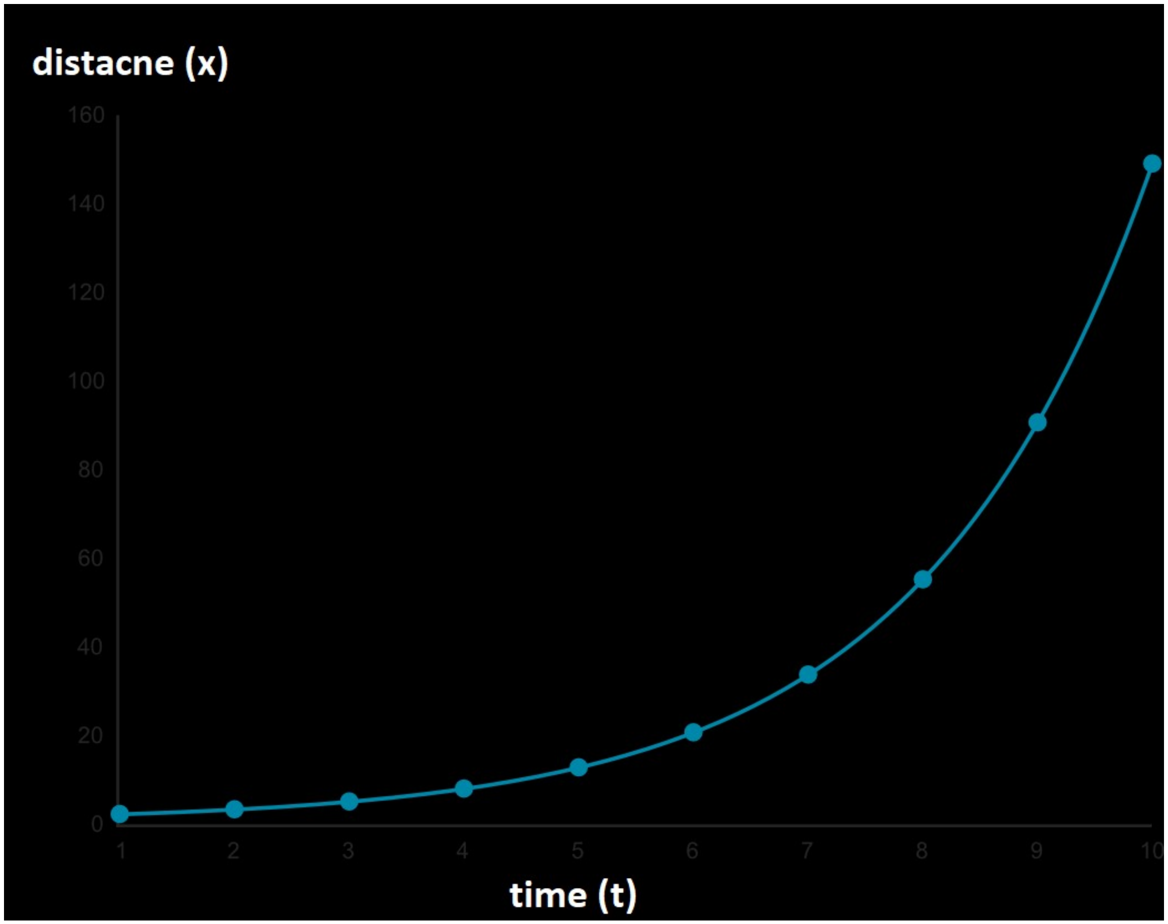
If a particle moving along a straight line travels unequal distances in equal intervals of time or equal distances in unequal interval of time, then the particle is moving with non-uniform speed.

Uniform Speed - Graph:



Non-uniform Speed - Graph:

Image source: https://teleport.org/

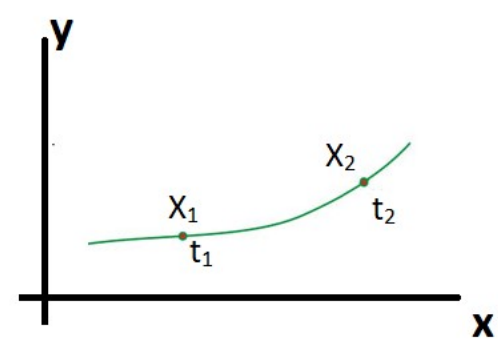


Average Speed:

For a particle in motion (uniform or non-uniform), the ratio of total distance travelled to the total time of motion is called average speed.

$$Average\ speed = \frac{Total\ distance\ travelled}{Total\ time}$$

$$= \frac{x_2 - x_1}{t_2 - t_1}$$



Where x_1 *and* x_2 *are the positions of an object at the instants* t_1 *and* t_2 *respectively*

If s_1, s_2,s_3,… s_n are the distances travelled by a particle in the time intervals t_1, t_2, t_3,… t_n respectively then,

$$Average\ Speed\ = \frac{s_1 + s_2 + s_3 + \ldots + s_n}{t_1 + t_2 + t_3 + \ldots + t_n}$$

Instantaneous Speed / Velocity:

The speed of a particle at a particular instant of time is called it's instantaneous speed. (or)

It is also defined as the limit of average speed as the time interval (Δt) becomes infinitesimally small.

If Δx is the distance travelled by a particle in a time interval Δt then

$$\text{speed} = \frac{\Delta x}{\Delta t}$$

VELOCITY

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Velocity:

The displacement of a body in unit time is called it’s velocity.

Uniform velocity:

If a particle moving equal displacements in equal intervals of time, then the particle is moving with uniform velocity.

Non-uniform velocity or Variable velocity:

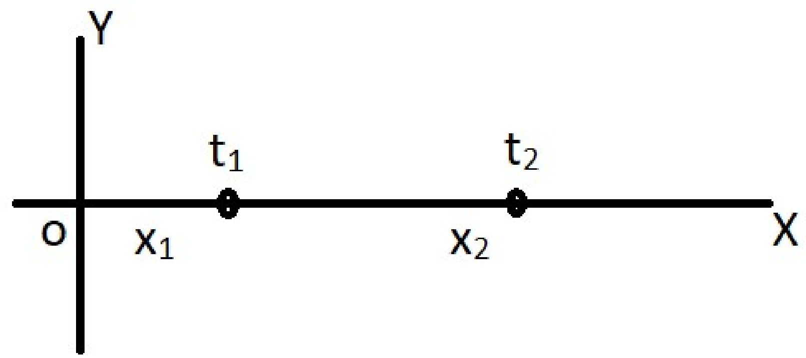
If a particle moving unequal displacements in equal intervals of time or equal displacements in unequal interval of time, then the particle is moving with non-uniform velocity.

Average Velocity:

For a particle in motion, the ratio of total displacement travelled to the total time of motion is called average velocity.

$$Average\ velocity = \frac{Total\ displacement\ travelled}{Total\ time}$$

$$= \frac{x_2 - x_1}{t_2 - t_1}$$



A body travelling between two positions traveles with speed v_1 for time t_1 and then with **velocity** v_2 for time t_2. For the total motion,

$$\text{Average } \textbf{Velocity} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$$

A body travelling between two positions travels for the time intervals $t_1, t_2, t_3 \ldots\ldots\ldots t_n$ with **velocity** $v_1, v_2, v_3, \ldots\ldots\ldots v_n$ respectively

$$Average\ velocity = \frac{Total\ displacement\ travelled}{Total\ time}$$

$$= \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \ldots\ldots\ldots v_n t_n}{t_1 + t_2 + t_3 + \ldots\ldots t_n}$$

ACCELERATION

Acceleration and Deceleration:

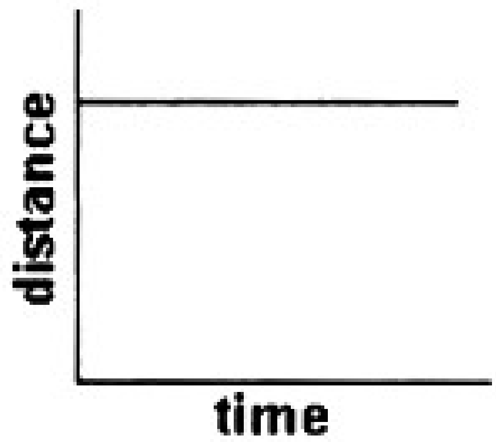
Acceleration is defined as the rate of change of velocity

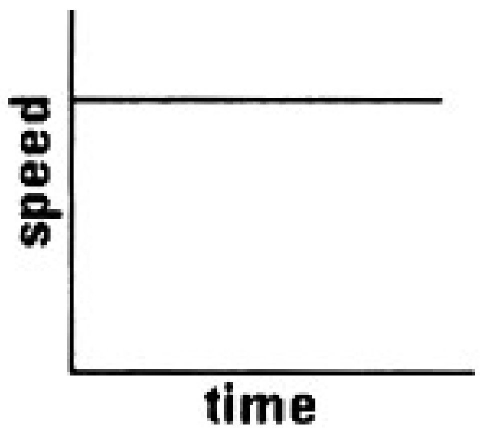
$$Let\ v_1, v_2\ be\ the\ velocities\ of\ a\ particle\ at\ instants\ t_1\ and\ t_2\ respectively$$

$$\boldsymbol{Acceleration = \frac{change\ in\ velocity}{time}}$$

$$\mathrm{a} = \frac{v_2 - v_1}{t_2 - t_1}$$

Graphical Analysis:

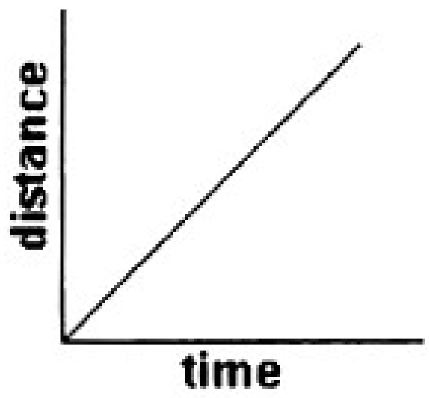


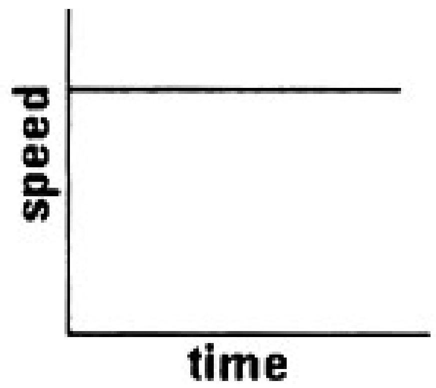


Particle is at rest

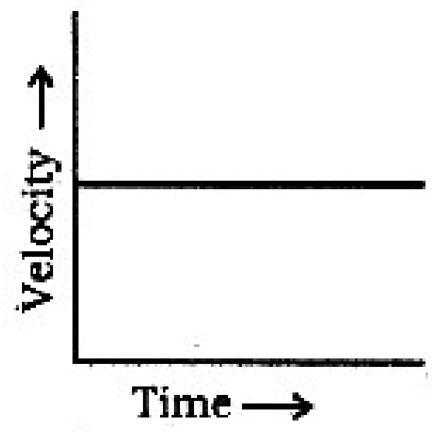
Uniform Velocity / Speed

Graphical Analysis:

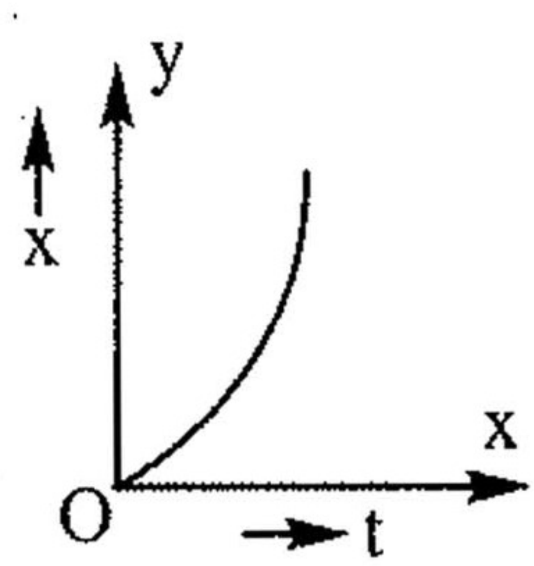




Uniform Velocity / Speed

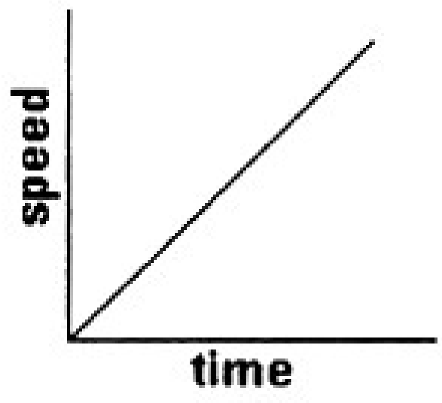


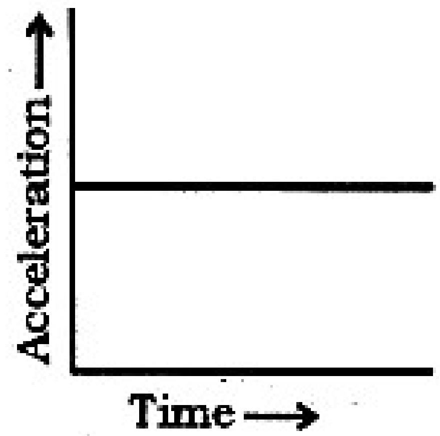
Graphical Analysis:



Non-uniform Velocity / Speed

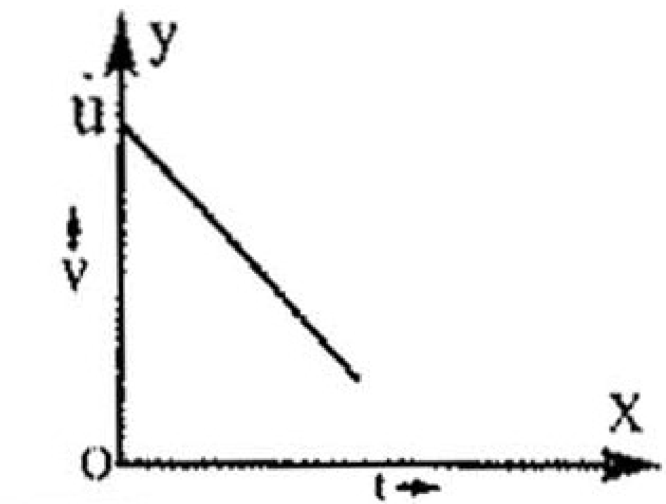
Graphical Analysis:





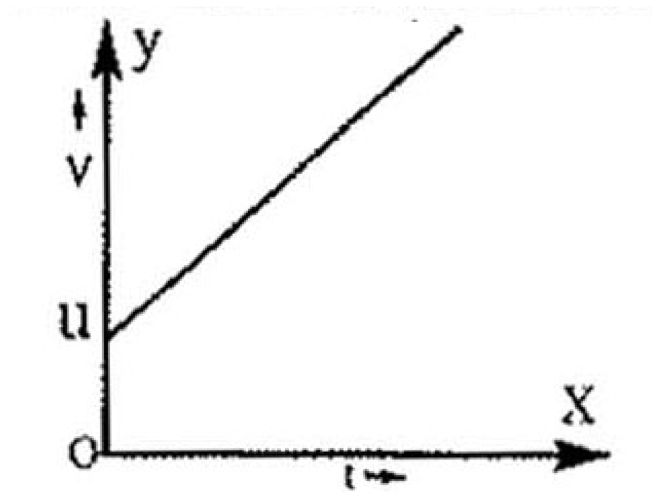
Uniform acceleration

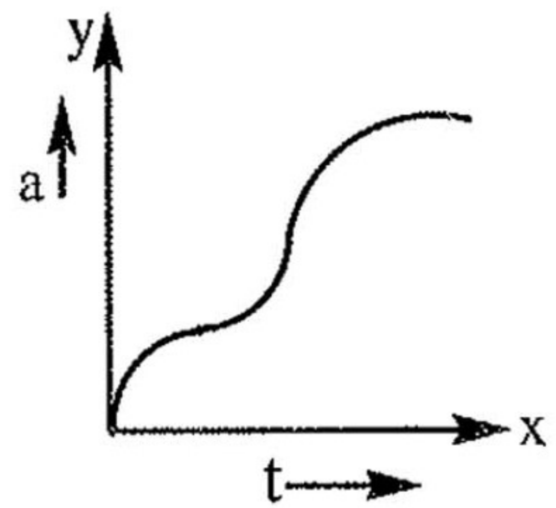
Graphical Analysis:



Uniform deceleration or retardation

Graphical Analysis:

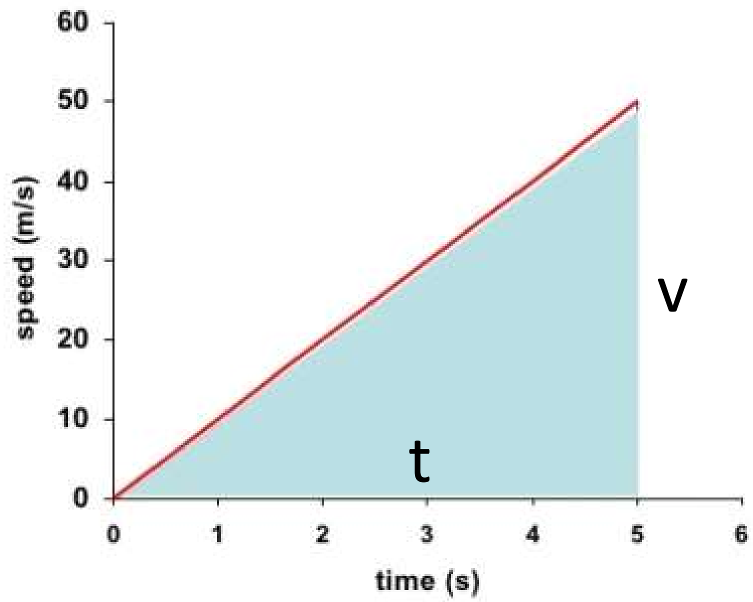




Uniform acceleration

Non-uniform acceleration

Graphical Analysis:



$$v = \frac{d}{t}$$

$$\Rightarrow \quad d = vt$$

Area under velocity – time curve gives the displacement of the particle or body.

DERIVATIONS

$$v = u + at$$

$$v^2 - u^2 = 2aS$$

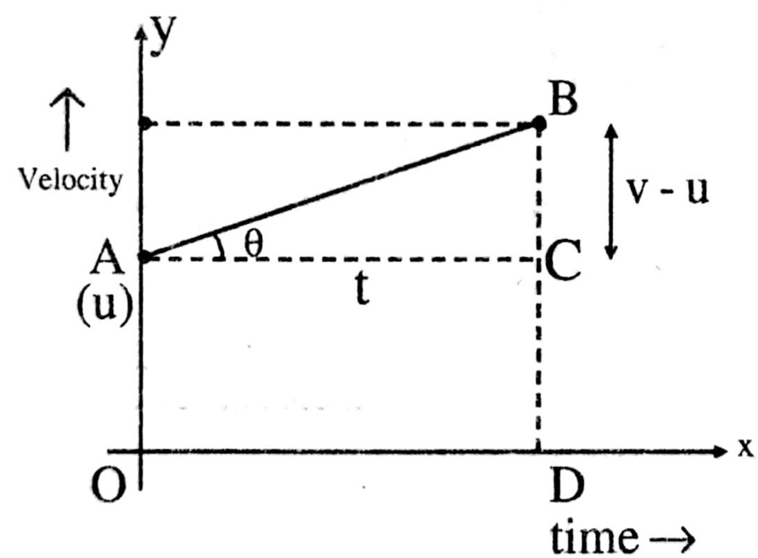
$$S = ut + \frac{1}{2}at^2$$

$$S_{n^{th}} = u + \frac{a}{2}(2n - 1)$$

Kinematical Equations of Motion of a Body with Uniform Acceleration: (Graphical treatment)

1) $v = u + at$ *derivation*:

The slope of the velocity-time graph gives the acceleration of the particle.



$$Slope = \tan\theta = \frac{BC}{AC}$$

$$\Rightarrow \quad a = \frac{v-u}{t}$$

$$\therefore\ v - u = at$$

$$\Rightarrow\ v = u + at \qquad \text{---------------} \quad (1)$$

2) $v^2 - u^2 = 2aS$ derivation:

When a particle is moving with uniform acceleration,

$$Average\ velocity = \frac{v+u}{2}$$

$$\therefore\ Displacement = Average\ velocity \times time$$

$$\therefore\ S = \left(\frac{v+u}{2}\right)(t)$$

$$\therefore\ S = \left(\frac{v+u}{2}\right)(t)$$

$$\Rightarrow\ v+u = \frac{2S}{t} \quad \text{--------------- (2)}$$

$$from\ equation\ (1)\ \ v = u + at$$

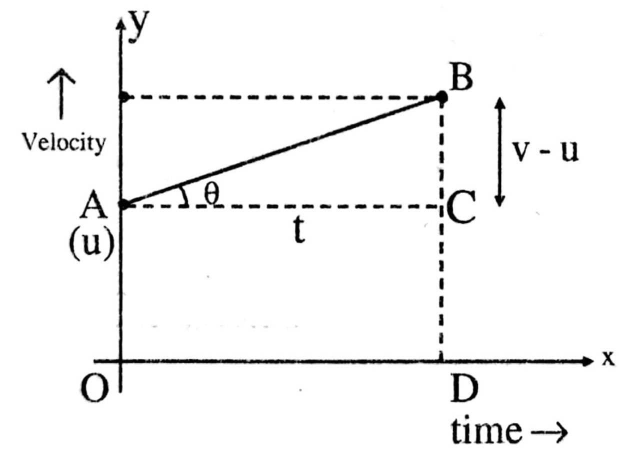
$$\Rightarrow\ v-u = at \quad \text{--------------- (3)}$$

Multiply equations (2) and (3), we get

$$\Rightarrow\ (v+u)(v-u) = \frac{2S}{t}at \ \Rightarrow\ \boxed{v^2 - u^2 = 2aS} \quad \text{------ (4)}$$

3) $S = ut + \frac{1}{2}at^2$ *derivation*:

The area under the velocity-time graph gives the displacement in the bounded time interval. Here the area bounded by the line AB with x-axis gives the displacement.



$$\therefore\ S = Area\ of\ rectangle\ OACD\ +\ Area\ of\ triangle\ ABC$$

$$\therefore\ S = (OA)(OD) + \frac{1}{2}(AC)(CB)$$

$$= (u)(t) + \frac{1}{2}(t)(v-u)$$

$$= (u)(t) + \frac{1}{2}(t)(v - u) \qquad from\ equation\ (3)\ v - u = at$$

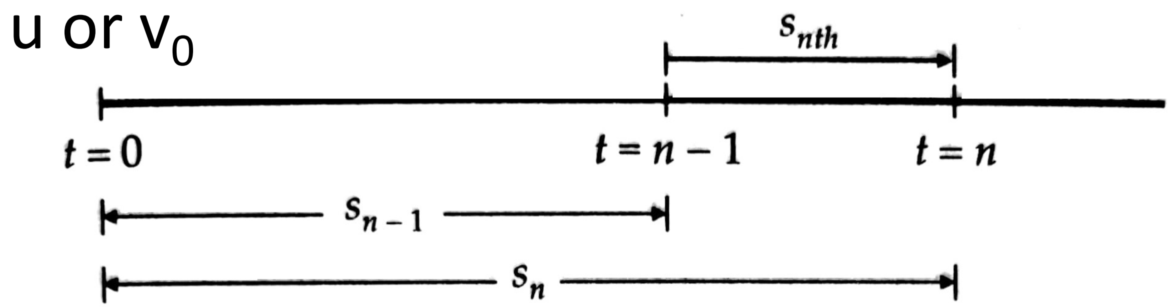
$$= ut + \frac{1}{2}(t)at$$

$$\therefore \quad S = ut + \frac{1}{2}at^2 \quad \text{--------------} \quad (5)$$

This is the expression for the distance travelled by the particle with initial velocity u and in time t.

4) $S_n = u + \frac{1}{2}a(2n - 1)$ *derivation*:

$$S = ut + \frac{1}{2}at^2$$



Distance travelled in first n seconds is

$$s_n = v_0 n + \frac{1}{2}an^2$$

Distance travelled in first $(n-1)$ seconds is

$$s_{n-1} = v_0(n-1) + \frac{1}{2}a(n-1)^2$$

Hence the distance travelled in nth second is

$$s_{nth} = s_n - s_{n-1}$$

$$= (v_0 n + \frac{1}{2} a n^2) - [v_0 (n-1) + \frac{1}{2} a (n-1)^2]$$

$$= (v_0 n + \frac{1}{2} a n^2) - [v_0 n - v_0 + \frac{1}{2} a (n^2 - 2n + 1)]$$

$$= v_0 n + \frac{1}{2} a n^2 - v_0 n + v_0 - \frac{1}{2} a n^2 + a n - \frac{a}{2}$$

$$s_{nth} = v_0 + \frac{a}{2}(2n - 1)$$

$$s_{nth} = v_0 + \frac{a}{2}(2n - 1)$$

$$S_{n^{th}} = u + \frac{a}{2}(2n - 1)$$

$$S_{n^{th}} = u + a\left(n - \frac{1}{2}\right)$$

Equations of motion, if acceleration a = g:

$$v = u + at$$

$$v = u + gt$$

$$v^2 - u^2 = 2aS$$

$$v^2 - u^2 = 2gS$$

$$S = ut + \frac{1}{2}at^2$$

$$S = ut + \frac{1}{2}gt^2$$

$$S_{n^{th}} = u + \frac{a}{2}(2n - 1)$$

$$S_{n^{th}} = u + \frac{g}{2}(2n - 1)$$

Equations of motion of a freely falling body:

For freely falling body initial velocity u = 0 and acceleration a = g

$$v = u + at \qquad\qquad v = gt$$

$$v^2 - u^2 = 2aS \qquad\qquad v^2 = 2gS$$

$$S = ut + \frac{1}{2}at^2 \qquad\qquad S = \frac{1}{2}gt^2$$

$$S_{n^{th}} = u + \frac{a}{2}(2n - 1) \qquad\qquad S_{n^{th}} = \frac{g}{2}(2n - 1)$$

Equations of motion for vertically projected body:

For freely falling body acceleration a = - g

$$v = u + at \Rightarrow v = u - gt$$

$$v^2 - u^2 = 2aS \Rightarrow v^2 = u^2 - 2gS$$

$$S = ut + \frac{1}{2}at^2 \Rightarrow S = ut - \frac{1}{2}gt^2$$

$$S_{n^{th}} = u + \frac{a}{2}(2n - 1) \Rightarrow S_{n^{th}} = u - \frac{g}{2}(2n - 1)$$

Maximum Height & Time of Flight

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Maximum Height (H_{max}):

For a body projected vertically upwards, the maximum vertical displacement from ground about which its velocity is zero is called its maximum height.

Let a body be projected vertically upwards with initial velocity u

We know that, $v^2 - u^2 = 2as$

here $a = -g, \quad s = H_{max}, \quad v = 0$

$\therefore \quad 0 - u^2 = 2(-g)H_{max}$

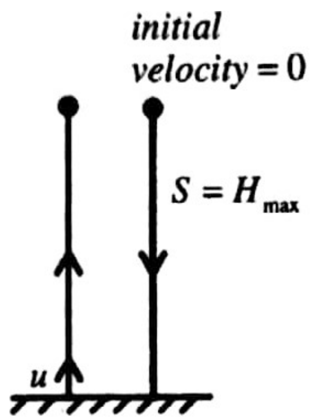
$\Rightarrow \quad -u^2 = -2gH_{max} \quad \Rightarrow \quad H_{max} = \dfrac{u^2}{2g}$

Time of ascent (t_a):

For a body projected upwards the time to reach the maximum height is called time of ascent.

Let a body be projected vertically upwards with initial velocity u.

$$We\ know\ that, \qquad v = u + at$$



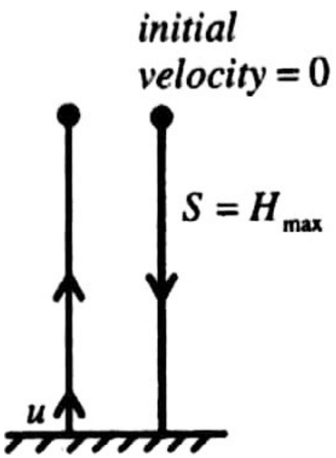
Here $a = -g, \quad t = t_a, \quad v = 0$

$$\therefore \quad 0 = u - gt_a \qquad \Rightarrow \quad gt_a = u$$

$$\Rightarrow \quad t_a = \frac{u}{g}$$

Time of descent (t_d):

For a body projected upwards the time to travel from maximum height to the point of projection on ground is called time of descent.



$$S = ut + \frac{1}{2}gt^2$$

initial velocity $= 0,\ a = +g,\ t = t_d,\ S = H_{max}$

$$\therefore\ H_{max} = 0 + \frac{1}{2}gt_d^2 \quad \Rightarrow \quad \frac{u^2}{2g} = \frac{1}{2}gt_d^2$$

$$H_{max} = \frac{u^2}{2g}$$

$$t_d^2 = \frac{u^2}{g^2} \quad \Rightarrow \quad \boxed{t_d = \frac{u}{g}}$$

Time of flight (T):

For a body projected vertically upwards, the sum of time of ascent and time of descent is called time of flight (T). It is the total time for which the body remains in air.

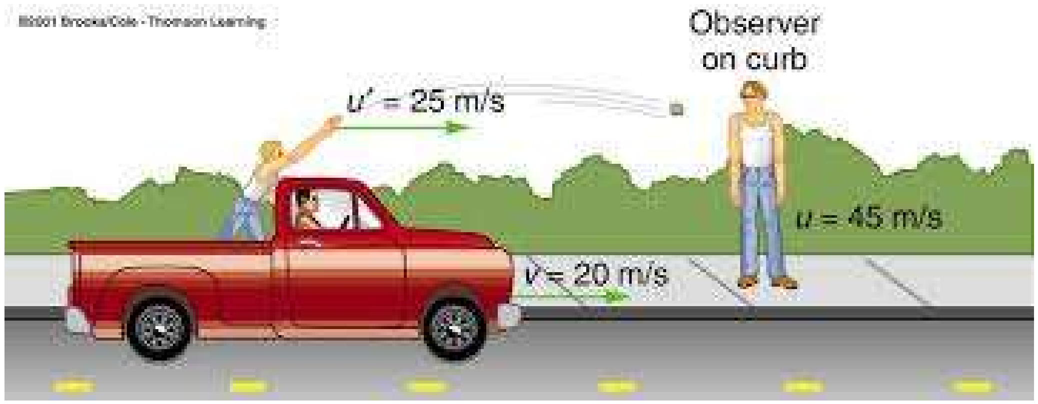
Time of flight = Time of ascent + Time of descent

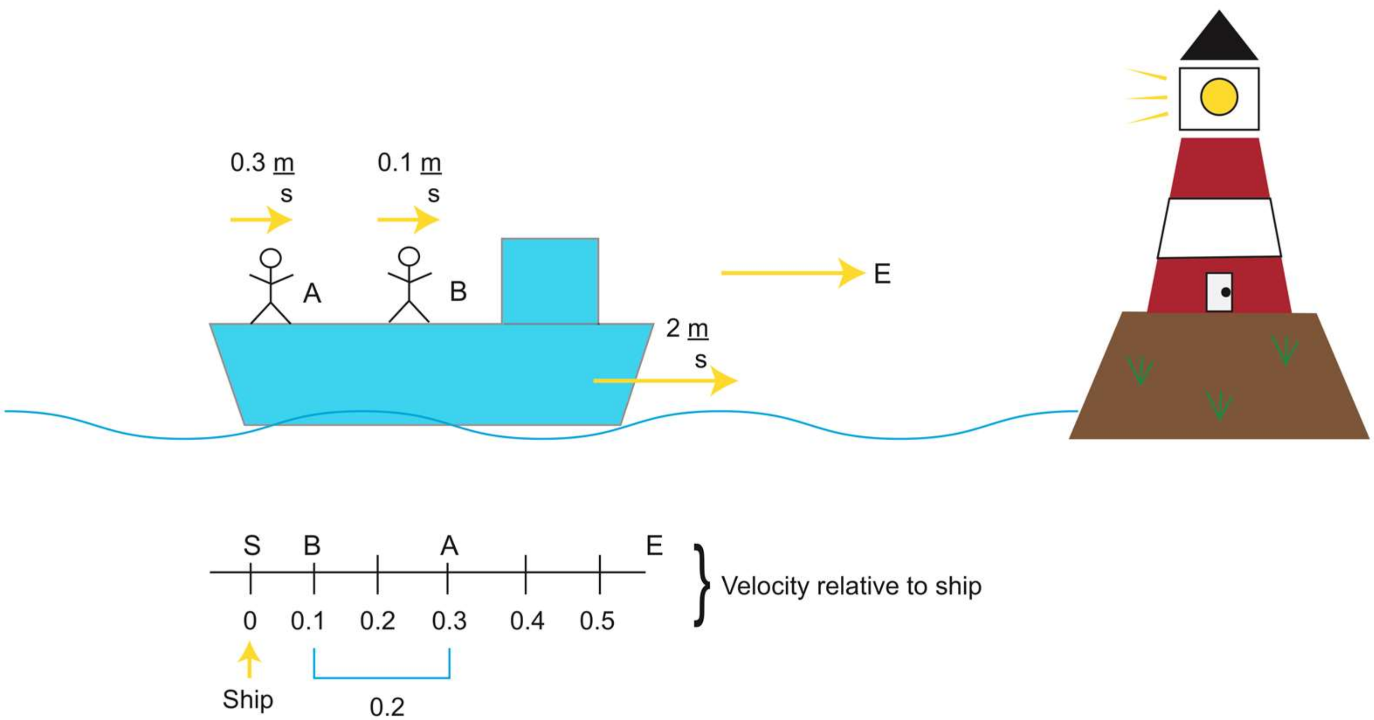
$$T = t_a + t_d$$

$$\therefore \quad T = \frac{u}{g} + \frac{u}{g}$$

$$\Rightarrow \quad T = \frac{2u}{g}$$

Relative velocity:





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