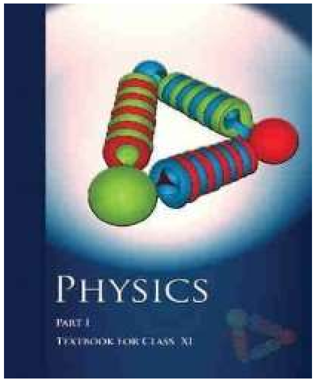


CLASS 11

CHAPTER - 8

GRAVITATION

P V S B CHALAPATHI,
Faculty in Physics.

GRAVITATION

SUB TOPICS :

NEWTONS LAW OF GRAVITATION

(GRAVITATIONAL FORCE)

KEPLER'S LAWS

ACCELERATION DUE TO GRAVITY

VARIATION OF ACCELERATION DUE TO GRAVITY WITH ALTITUDE AND DEPTH

ORBITAL AND ESCAPE VELOCITIES

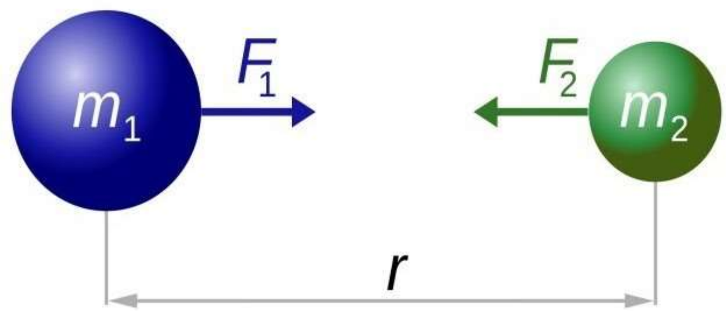
GRAVITATIONAL POTENTIAL ENERGY

SATELLITE

NEWTON’S LAWS OF MOTION (GRAVITATIONAL FORCE)

Gravitational force

Gravitational force is the force of mutual attraction between any two massive bodies by virtue of their masses

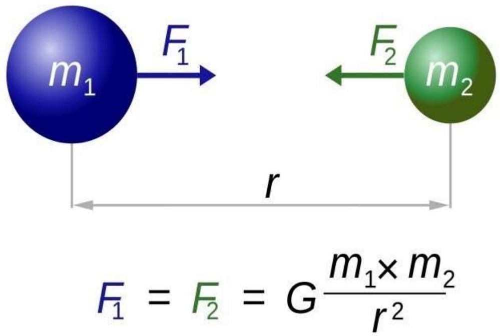


$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Where G = 6.67 × 10^{-11} Nm^2/kg^2

Image source: en.wikipedia.org

PROPERTIES:



Gravitational force is

- Very weak in nature
- Long-range forces
- Independent of medium between them
- Depends upon the product of the magnitude of the masses
- Inversely proportional to the distance between the masses
- Always along the line joining the two masses
- Only attractive

KEPLER’S LAWS
OF
PLANETARY MOTION

KEPLER'S LAWS OF PLANETARY MOTION

Three laws

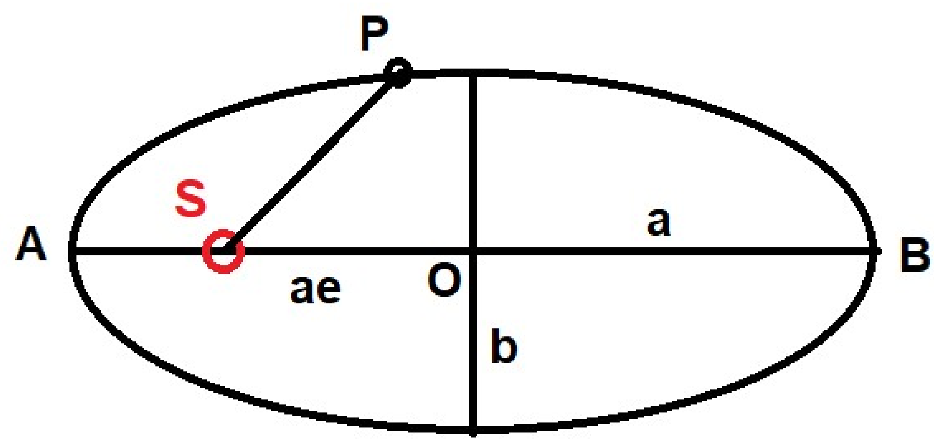
1st law – Law of orbits

2nd law – Law of areas

3rd law – Law of periods

Kepler's I law:

Every planet revolves around the sun in an elliptical orbit with the sun is at one of its foci. This law is also known as the law of Orbits.



Kepler's II law:

The radius vector drawn from the sun to the planet gives equal areas in equal interval of time. That means the rate of change of the area of the planet / areal velocity is constant. This law is also known as the law of areas.

$$dA/dt = \text{Constant}$$

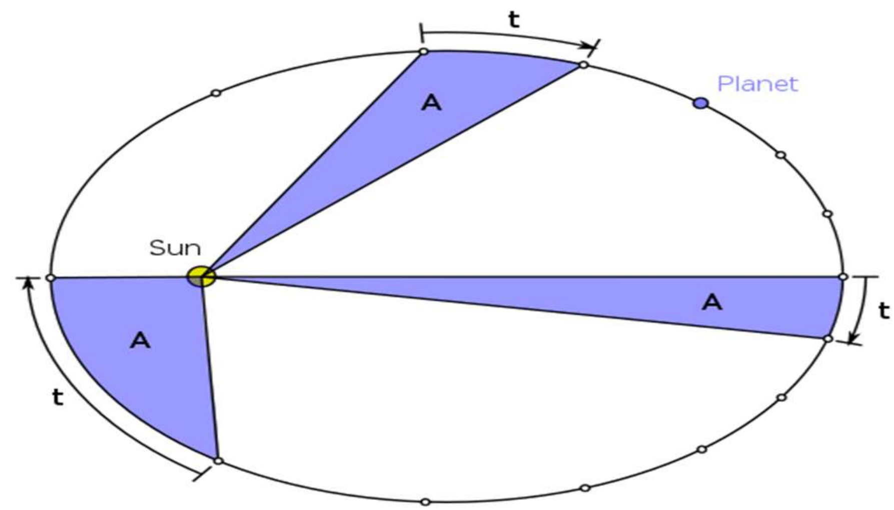


Image source: www.commons.wikimedia.org

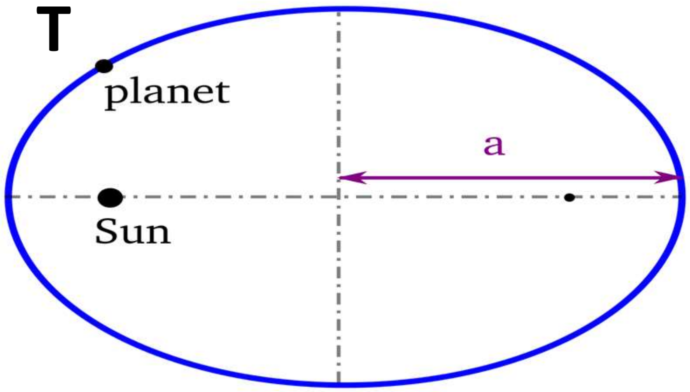
Kepler's III law:

The square of the time period (T) of the planet revolving around the sun is directly proportional to the cube of the semi-major axis (a) of the orbit. This law is also known as the law of periods.

i.e $T^2 \propto a^3$

$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

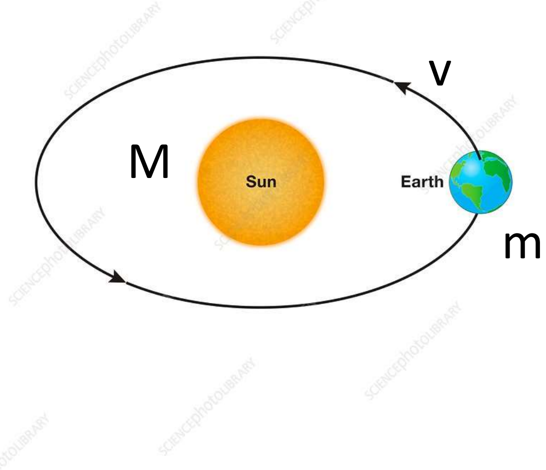
T



Or r

KEPLER III LAW PROOF:

Image source: www.sciencephoto.com



Centrifugal force = $\dfrac{mv^2}{R}$

Centripetal force $= \dfrac{GMm}{R^2}$

Centrifugal force = Centripetal force

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

$$\Rightarrow \frac{GM}{R} = v^2$$

Speed of the planet is $v = \frac{2\pi R}{T}$

$\frac{GM}{R} = v^2$

$$\frac{GM}{R} = \left[\frac{2\pi R}{T}\right]^2 = \frac{4\pi^2 R^2}{T^2}$$

$$\Rightarrow T^2 = \frac{4\pi^2}{GM} R^3$$

$$\Rightarrow T^2 \propto R^3$$

ACCELERATION DUE TO GRAVITY

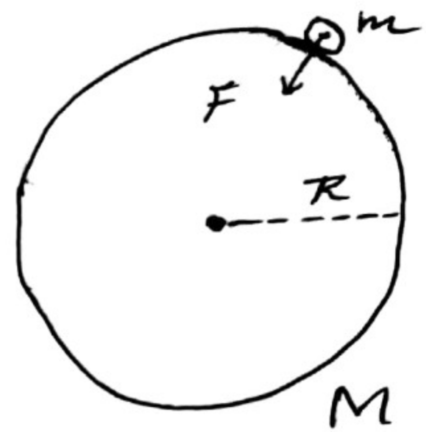
ACCELERATION DUE TO GRAVITY:

$$F = mg \longrightarrow (1)$$

$$F = \frac{GMm}{R^2} \longrightarrow (2)$$

$$mg = \frac{GMm}{R^2}$$

$$\Rightarrow g = \frac{GM}{R^2}$$



Acceleration due to gravity in terms of radius (R) and density (ρ)

Density $\rho = \dfrac{M}{V}$

$$\Rightarrow M = \rho \cdot V$$

$$\Rightarrow M = \rho\left[\frac{4}{3}\pi R^3\right]$$

$$g = \frac{GM}{R^2}$$

$$\therefore g = \frac{G\rho\left[\frac{4}{3}\pi R^3\right]}{R^2}$$

$$\Rightarrow g = \frac{4}{3}\pi G \rho R$$

Acceleration due to gravity in terms of mass (M) and density (ρ)

$$M = \rho\left[\frac{4}{3}\pi R^3\right]$$

$$R^3 = \frac{3M}{4\pi\rho}$$

$$\Rightarrow R = \left[\frac{3M}{4\pi\rho}\right]^{1/3}$$

$$\Rightarrow \quad R^2 = \left[\frac{3M}{4\pi\rho}\right]^{2/3}$$

$$= \left[\frac{3}{4\pi}\right]^{2/3} \frac{M^{2/3}}{\rho^{2/3}}$$

$$\therefore \; g = \frac{GM}{R^2}$$

$$= \frac{GM}{\left(\frac{3}{4\pi}\right)^{2/3} \frac{M^{2/3}}{\rho^{2/3}}}$$

$$= \frac{GM}{\left(\frac{3}{4\pi}\right)^{2/3} \frac{M^{2/3}}{\rho^{2/3}}}$$

$$= \left[\frac{4\pi}{3}\right]^{2/3} G \cdot M^{1-\frac{2}{3}} \cdot \rho^{2/3}$$

$$\therefore g = \left[\frac{4\pi}{3}\right]^{2/3} G M^{1/3} \rho^{2/3}$$

Variation of value of Acceleration due to Gravity with Altitude(Height)

Variation of 'g' with altitude(height)

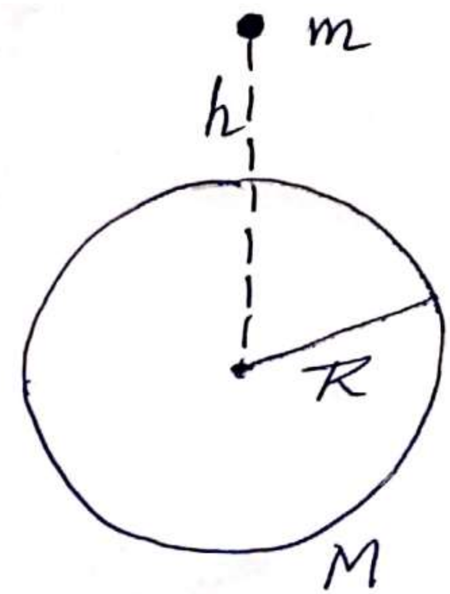
$$r = R + h$$

According to Newton's law of gravitation

$$F = \frac{GMm}{r^2}$$

$$\Rightarrow F = \frac{GMm}{(R+h)^2} \longrightarrow (1)$$

$$F = mg_h \longrightarrow (2)$$



From eqns 1 & 2

$$mg_h = \frac{GMm}{(R+h)^2}$$

$$\Rightarrow g_h = \frac{GM}{(R+h)^2} \longrightarrow 3$$

$$g_h = \frac{GM}{\left[R\left[1+\frac{h}{R}\right]\right]^2}$$

$$= \frac{GM}{R^2\left[1+\frac{h}{R}\right]^2}$$

$$= \frac{GM}{R^2\left[1+\frac{h}{R}\right]^2}$$

We know that $g = \frac{GM}{R^2}$

$$\Rightarrow g_h = \frac{g}{\left[1+\frac{h}{R}\right]^2}$$

For smaller heights

$$g_h = \frac{g}{\left[1+\frac{h}{R}\right]^2}$$

$$g_h = g\left[1+\frac{h}{R}\right]^{-2}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(1+x)^2 = 1 + 2x + x^2$$

$$(1+x)^2 = 1 + 2x$$

$$(1+x)^3 = 1 + 3x + 3x^2 + x^3$$

$$\Rightarrow (1+x)^3 = 1 + 3x$$

$$(1+x)^n = 1 + nx$$

$$(1+x)^n = 1 + nx$$

$$g_h = g\left[1 + \frac{h}{R}\right]^{-2}$$

$$\therefore g_h = g\left[1 - \frac{2h}{R}\right]$$

Acceleration due to gravity decreases as height increases

Variation of value of Acceleration due to Gravity with Depth

Variation of ‘g’ with depth:

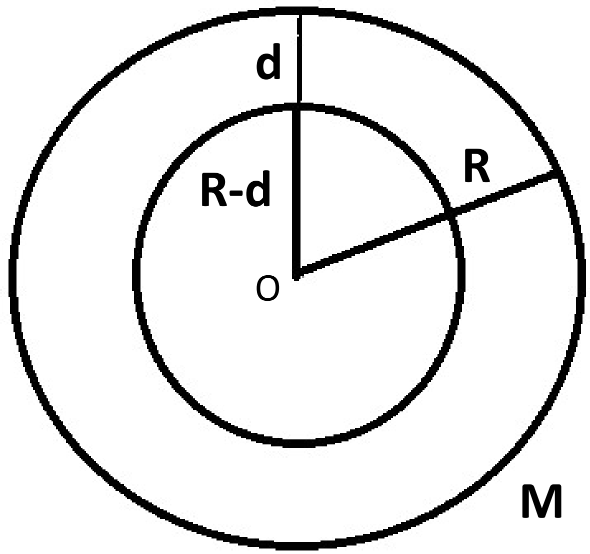
We know that Acceleration due to gravity

$$g = \frac{GM}{R^2}$$

Acceleration due to gravity in terms of Radius and Density

$$g = \frac{4\pi}{3} G \rho R$$

$$g_d = \frac{4\pi}{3} G \rho (R-d)$$



$$g_d = \frac{4}{3}\pi G \rho (R - d)$$

$$= \frac{4}{3}\pi G \rho R \left[1 - \frac{d}{R}\right]$$

$$\therefore g_d = g \left[1 - \frac{d}{R}\right]$$

The Value of acceleration due to gravity decreases with the depth

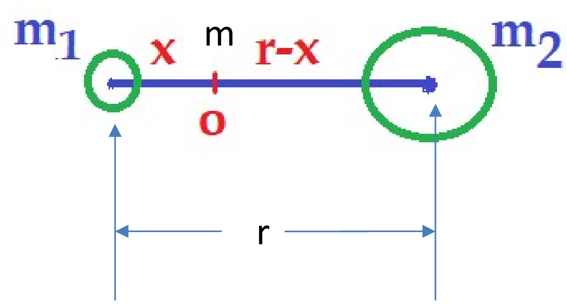
NULL POINTS

IN

GRAVITATIONAL FIELD

NULL POINT

The point at which the resultant gravitational force is zero is known as null point.



$$F_1 = \frac{G m_1 m}{x^2}$$

$$F_2 = \frac{G m_2 m}{(r-x)^2}$$

$$F_1 = F_2$$

$$\frac{G m_1 m}{x^2} = \frac{G m_2 m}{(r-x)^2}$$

$$\Rightarrow \frac{m_1}{x^2} = \frac{m_2}{(r-x)^2}$$

$$\Rightarrow \frac{(r-x)^2}{x^2} = \frac{m_2}{m_1}$$

$$\Rightarrow \left(\frac{r-x}{x}\right)^2 = \frac{m_2}{m_1}$$

$$\Rightarrow \frac{r-x}{x} = \sqrt{\frac{m_2}{m_1}}$$

$$\Rightarrow \frac{r}{x} - 1 = \sqrt{\frac{m_2}{m_1}}$$

$$\Rightarrow \frac{r}{x} = \sqrt{\frac{m_2}{m_1}} + 1$$

X – distance from smaller mass

$$\Rightarrow x = \frac{r}{\sqrt{\frac{m_2}{m_1}} + 1}$$

Two masses 2kg and 8kg are 12cm apart. Find the distance of the point where net gravitational force is zero from the smaller mass.

Given

$m_1 = 2kg$

$m_2 = 8kg$

$r = 12\ cm$

$$x = \frac{r}{\sqrt{\frac{m_2}{m_1}}+1}$$

$$= \frac{12}{\sqrt{\frac{8}{2}}+1}$$

$$= \frac{12}{\sqrt{4}+1}$$

$$= \frac{12}{2+1}$$

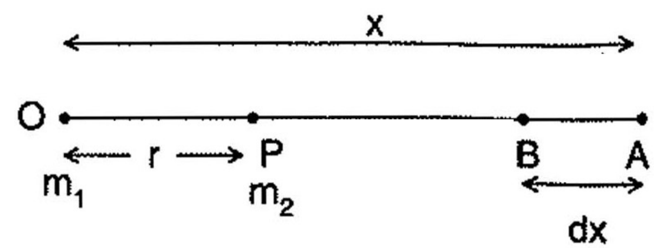
$$= \frac{12}{3}$$

$$= 4\ cm$$

GRAVITATIONAL POTENTIAL ENERGY

Gravitational Potential Energy

Potential energy of a system of two masses is defined as the amount of work done in bringing these two masses from infinity to their respective places



$$F = \frac{Gm_1m_2}{x^2}$$

$$dW = \vec{F}.\overrightarrow{dx} = F\ dx\cos\theta$$

$$dW = F\,dx \qquad (\because \theta = 0)$$

$$dW = \frac{Gm_1m_2}{x^2}dx$$

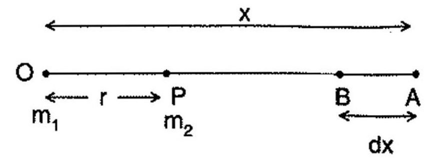
$$\therefore\ W = \int dW = \int_{\infty}^{r} \frac{Gm_1m_2}{x^2}dx$$

$$= Gm_1m_2 \int_{\infty}^{r} x^{-2}\,dx$$

$$= -Gm_1m_2\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$= -Gm_1m_2\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$= \frac{-Gm_1m_2}{r}$$



$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$ **(Integration)**

(Differentiation) $$\frac{d(x^n)}{dx} = nx^{n-1}$$

$$\int x^2\,dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3}$$

$$\int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} = \frac{-1}{x}$$

$$= \frac{-Gm_1m_2}{r}$$

The work done is stored in the particle as its gravitational potential energy (U).

Therefore gravitational potential energy of the particles

$$U = \frac{-Gm_1m_2}{r}$$

ESCAPE VELOCITY

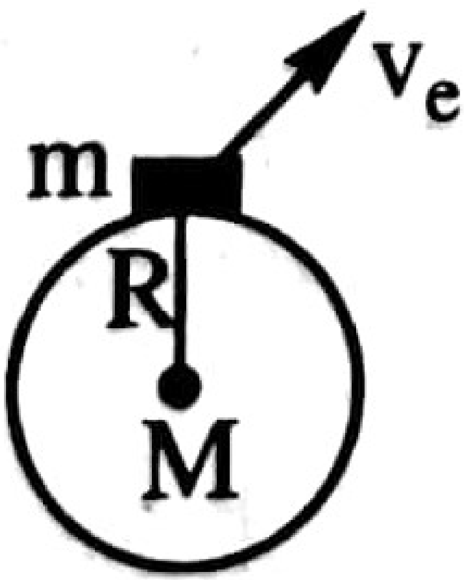
Escape Velocity:

The minimum velocity required for a projectile to escape from the surface of a planet (from gravitational influence of a planet) is called escape velocity of that planet.

$$\therefore PE + KE = 0$$

$$\Rightarrow KE = -\ PE$$

$$\Rightarrow \frac{1}{2}mv_e^2 = -\left(\frac{-GMm}{R}\right)$$



$$\Rightarrow \frac{1}{2} m v_e^2 = \frac{GMm}{R}$$

$$\Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

In terms of g,

$$v_e = \sqrt{\frac{2GM\ R}{R^2}}$$

$$\because g = \frac{GM}{R^2}$$

$$v_e = \sqrt{2gR}$$

Where g = *9.8m/s²*

R= *6.4 × 10⁶m*

$$\Rightarrow V_e = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

On the surface of earth

v_e=11.2km/s

Escape velocity of a body from certain height above the surface of a planet

At a particular height h, the distance from the center of the planet = R + h

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$v_e = \sqrt{2gR}$$

$$\boxed{V_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{2g_h(R+h)}}$$

ORBITAL VELOCITY

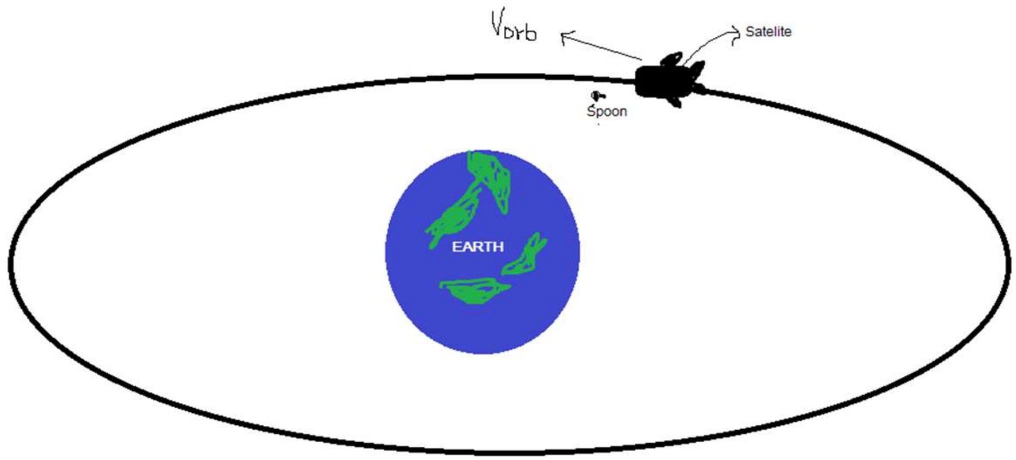


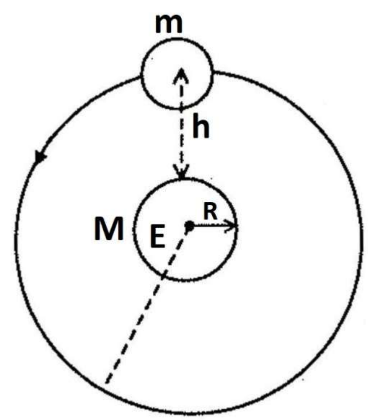
Image source: physics.stackexchange.com

Orbital Velocity:

Definition:

Velocity of a satellite moving in the orbit is called orbital velocity.

Radius of the satellite = R + h



Centrifugal force = $\frac{mV^2}{r} = \frac{mV_0^2}{R+h}$

Centripetal force = $\frac{GM \cdot m}{(R+h)^2}$

Centrifugal force = Centripetal force

$$\frac{mV_0^2}{R+h} = \frac{GM \cdot m}{(R+h)^2}$$

$$\therefore \quad v_o^2 = \frac{GM}{R+h}$$

$$\Rightarrow \quad v_o = \sqrt{\frac{GM}{R+h}}$$

Note:

Orbital velocity independent of mass of the satellite

Special case:

$$\text{If} \quad h << R, \quad v_o = \sqrt{\frac{GM}{R}}$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{GM \cdot R}{R \cdot R}}$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{GM}{R^2} \cdot R}$$

$$\boxed{\therefore \quad v_0 = \sqrt{gR}}$$

We know that

$$V_e = \sqrt{2gR}$$

$$\therefore V_e = \sqrt{2} \cdot \sqrt{gR}$$

$$\Rightarrow \boxed{V_e = \sqrt{2}\, V_o}$$

Escape velocity is greater than Orbital velocity

ARTIFICIAL SATELLITE

SATELLITE

A satellite is an object that moves around a larger object.

- Earth is a satellite because it moves around the sun.
- The moon is a satellite because it moves around Earth.
- Earth and the moon are called "natural" satellites

ARTIFICIAL SATELLITE

Artificial satellites are man-made bodies that revolve around the earth. They are used for multiple purposes like communication, weather forecasting, remote sensing, etc.

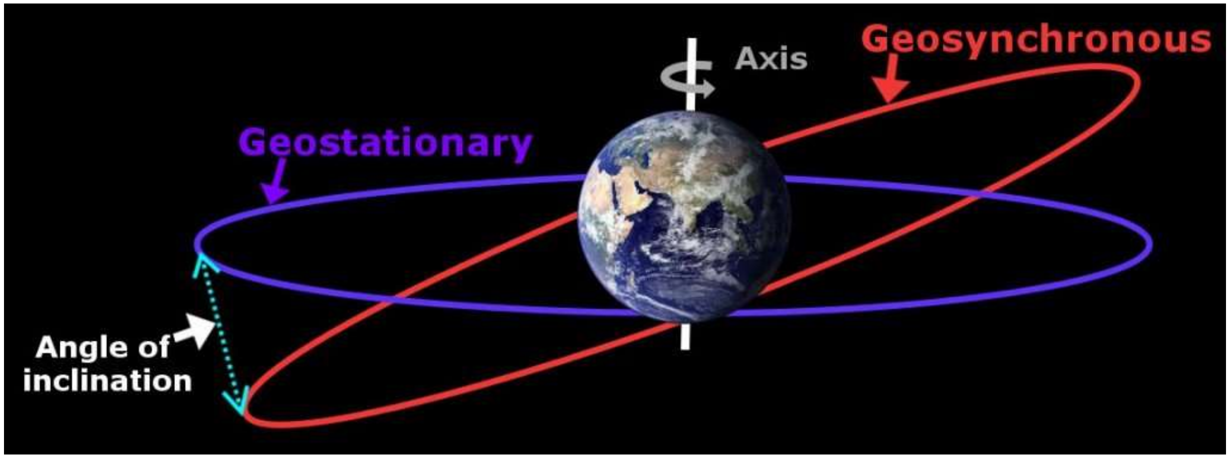
- Geostationary Satellite
- Polar Satellite

Geosynchronous Satellite Launch Vehicle (GSLV)

Polar Satellite Launch Vehicle (PSLV)

Geostationary Satellite

If the period of revolution of an artificial satellite is equal to the period of rotation of earth, then such a satellite is called geostationary satellite.



Conditions for Geostationary Satellite

- Height of the orbit above the earth surface is about 36,000 km
- The orbit of satellite must lie in the equatorial plane of the earth
- The sense of the revolution must be same as that of the rotation of the about its own axis
- Must move from west to east in the equatorial plane of the earth
- Time period of revolution of the satellite must be equal to time period of the earth's rotation i.e. 24 hr.
- The orbital angular velocity (ω) of geostationary satellite should be same as that of the earth

Uses of geostationary satellites

- For the study of the upper layers of the atmosphere
- For forecasting the changes in atmosphere and weather
- For finding the size and shape of the earth
- For investigating minerals and ores present in the earth's crust
- For transmission of T.V. signals
- For navigation – including the Global Positioning System (GPS)
- For space research

Polar Satellite

Polar satellite is a satellite whose orbit is perpendicular or at right angles to the equator

or

A satellite passes over the north and south poles as it orbits the earth.

It can be at any height from the earth, typically at 500–800 Kms

The time period of polar satellite is around 100 minutes. it's times period is about 100 mins., as a result it crosses any altitude many times within a day

FORMULAE :

$$g = \frac{GM}{R^2} = \left[\frac{4\pi}{3}\right]^{2/3} G M^{1/3} \rho^{2/3}$$

$$F = \frac{GMm}{R^2}$$

$T^2 \alpha a^3$

$$g = \frac{4}{3}\pi G \rho R$$

$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

$$g_h = \frac{g}{\left[1+\frac{h}{R}\right]^2}$$

$$T^2 = \frac{4\pi^2}{GM} R^3$$

$$g_h = g\left[1 - \frac{2h}{R}\right]$$

$$g_d = g\left[1 - \frac{d}{R}\right]$$

$$\Rightarrow x = \frac{r}{\sqrt{\frac{m_2}{m_1}} + 1}$$

FORMULAE :

$$U = \frac{-Gm_1m_2}{r}$$

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$v_e = \sqrt{2gR}$$

$$\boxed{Ve = \sqrt{\frac{2GM}{R+h}} = \sqrt{2g_h(R+h)}}$$

$$v_o = \sqrt{\frac{GM}{R+h}}$$

$$\boxed{\therefore\ v_o = \sqrt{gR}}$$