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MOVING CHARGES AND MAGNETISM

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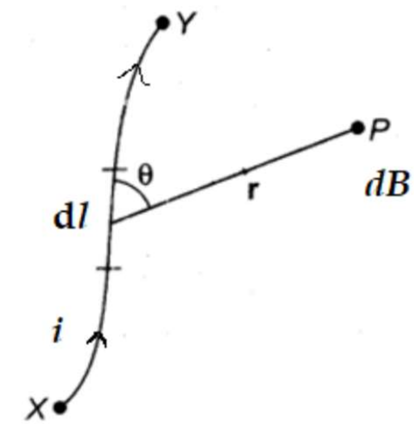
Biot-Savart's law:

According to Biot and Savart,



1) Directly proportional to the current '*i* ' flowing through the conductor

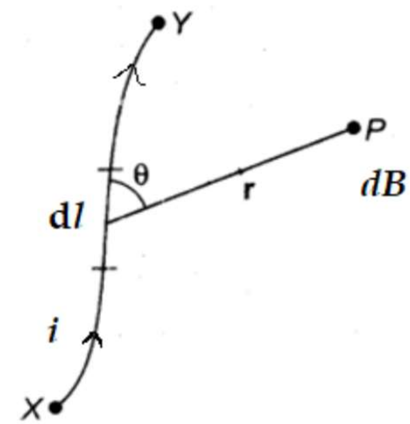
$$dB \propto i$$

2) Directly proportional to the length element dl

$$dB \propto dl$$

3) Directly proportional to the sine of the angle between length element and the line joining the element to the point P

$$dB \ \alpha \ \sin \Theta$$



4) Inversely proportional to the square of the distance r of the point P from the length element dl

$$dB \ \alpha \ \frac{1}{r^2}$$

$$dB \ \alpha \ \frac{idl\sin\theta}{r^2}$$

$$\Rightarrow \qquad dB = \frac{\mu_0}{4\pi} \frac{idl\sin\theta}{r^2}$$

Where $\frac{\mu_0}{4\pi}$ *is proportionality constant and*

μ_0 is the permeability of free space.

$$\frac{\mu_0}{4\pi} = 10^{-7} \ \frac{weber}{ampere\ meter}$$

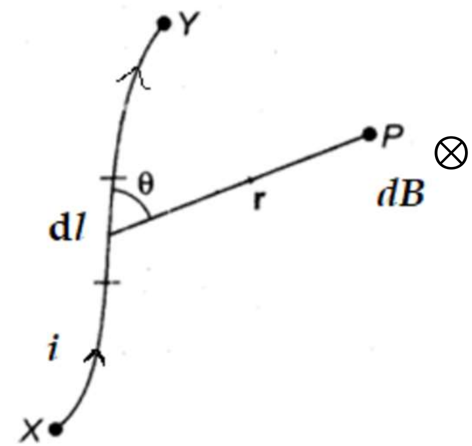
In vector form

$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{idl\sin\theta}{r^2} \hat{r}$$

$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{idl\sin\theta}{r^2} \frac{\bar{r}}{r}$$

$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{i(d\bar{l}\, X\, \bar{r})}{r^3}$$

$$\boldsymbol{dB} = \frac{\mu_0}{4\pi} \frac{i\,(\boldsymbol{dl}\, X\, \boldsymbol{r})}{r^3}$$



Ampere's right hand thumb rule / Ampere's law:

When a straight conductor carrying current is held in the right hand such that the thumb is pointing along the direction of current, then the direction in which fingers curl round it gives the direction of magnetic lines of force.

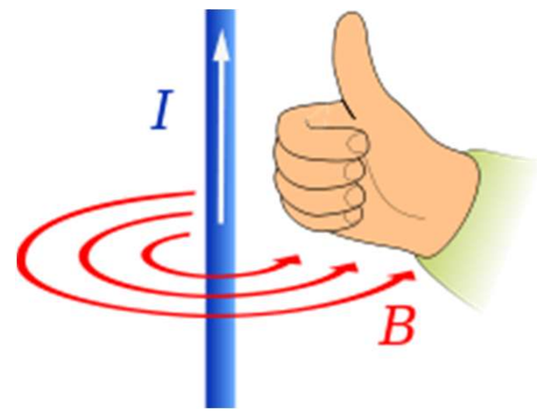


Image source: https://en.wikipedia.org

Mathematically, $$\oint \bar{B}.d\bar{l} = \mu_0 i$$

Magnetic Induction at a Point due to finite and infinite Straight Current Carrying Conductor

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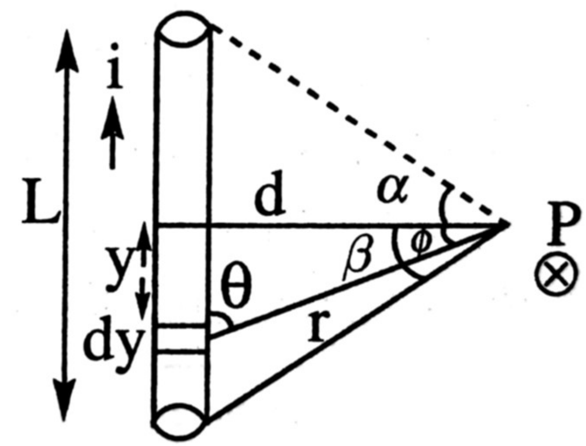
Application of Biot-Savart's law:

Magnetic induction due to a straight current carrying wire:

According to Biot – Savart's law, the magnetic induction at point P due to the small length element dy is



$$dB = \frac{\mu_0}{4\pi}\frac{idy\sin\theta}{r^2}$$

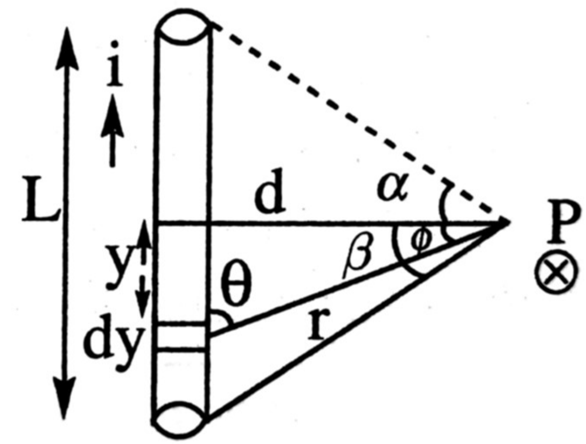
$\therefore$ *The magnetic induction due to the entire conductor is*

$$B = \int dB = \frac{\mu_0 i}{4\pi}\int\frac{dy\sin\theta}{r^2}$$

From the diagram

$$B = \frac{\mu_0 i}{4\pi}\int \frac{dy\sin\theta}{r^2}$$

$$\tan\phi = \frac{y}{d}$$



$$\Rightarrow \qquad y = d\tan\phi$$

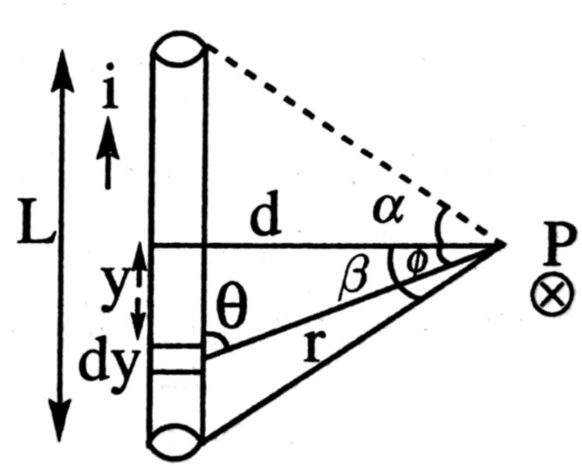
$$dy = d\,(\sec^2\phi)d\phi$$

From the diagram

$$\sec\phi = \frac{r}{d} \quad \Rightarrow \quad r = d\sec\phi$$

$$B = \frac{\mu_0 i}{4\pi}\int \frac{d(\sec^2\phi)d\phi\sin\theta}{d^2\sec^2\phi}$$

$$B = \frac{\mu_0 i}{4\pi} \int \frac{d(\sec^2\phi)\, d\phi\, \sin\theta}{d^2 \sec^2\phi} \qquad \because\ \theta = 90^0 - \phi$$



$$\Rightarrow \quad B = \frac{\mu_0 i}{4\pi} \int \frac{d\phi\, \sin(90^0 - \phi)}{d}$$

$$\Rightarrow \quad B = \frac{\mu_0 i}{4\pi} \int_{-\beta}^{\alpha} \frac{d\phi\, cos\phi}{d}$$

$$\Rightarrow \qquad B = \frac{\mu_0 i}{4\pi d} \int_{-\beta}^{\alpha} \cos\phi \, d\phi$$

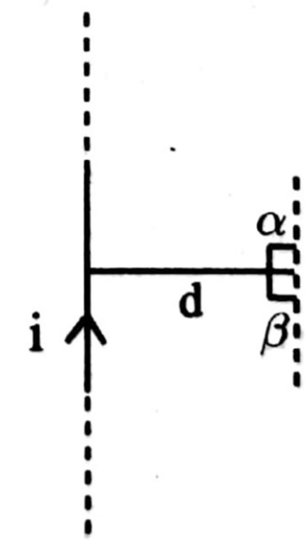
$$\Rightarrow \qquad B = \frac{\mu_0 i}{4\pi d} \left[\sin\phi\right]_{-\beta}^{\alpha}$$

$$\Rightarrow \qquad B = \frac{\mu_0 i}{4\pi d} [\sin\alpha - \sin(-\beta)]$$

$$\Rightarrow \qquad B = \frac{\mu_0 i}{4\pi d}[\sin\alpha + \sin\beta]$$

Special case:

If the wire is of infinite length and the point P lies at a distance d from the wire as shown in figure

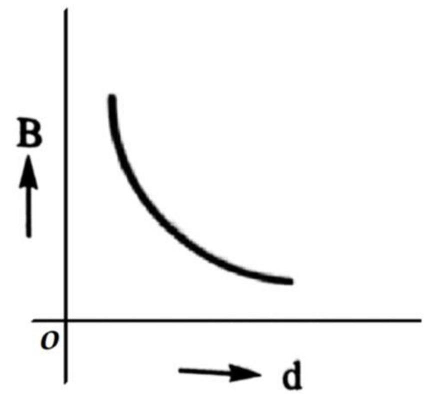


$$B = \frac{\mu_0 i}{4\pi d}[\sin 90^0 + \sin 90^0]$$

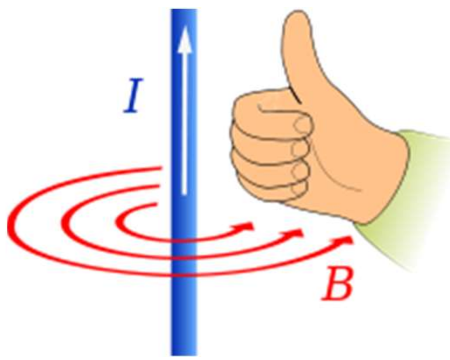
$$\Rightarrow \qquad B = \frac{\mu_0 i}{2\pi d}$$

$$B = \frac{\mu_0 i}{2\pi d}$$

$$\therefore \quad B \; \alpha \; \frac{1}{d}$$



Ampere’s law derivation:



$$B = \frac{\mu_0 i}{2\pi l}$$

$$B(2\pi l) = \mu_0 i$$

$$\oint \bar{B}.d\bar{l} = \mu_0 i$$

Problem: An infinitely long conductor carries a current of 100 mA. Find the magnetic induction at a point 10 cm away from it.

Solution:

Given

current i = 100 mA

= 100 x 10^{-3} A

= 10^{-1} A

distance d = 10 cm

= 10 x 10^{-2} m

= 10^{-1} m

$$B = \frac{\mu_0 i}{2\pi d}$$

$$= \frac{4\pi \; X \; 10^{-7} \; X \; 10^{-1}}{2\pi \; X \; 10^{-1}}$$

$$= 2 \; X \; 10^{-7} \text{ tesla}$$

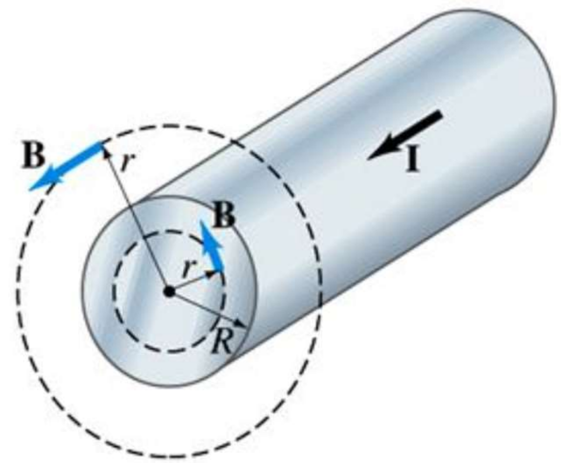
Magnetic induction inside a current carrying conductor:

$$Current\ density\ j = \frac{i}{A}$$

$$j_{enclosed} = j$$

$$\Rightarrow \quad \frac{i_{enclosed}}{\pi r^2} = \frac{i}{\pi R^2}$$

$$\Rightarrow \quad i_{enclosed} = \frac{i\, r^2}{R^2}$$



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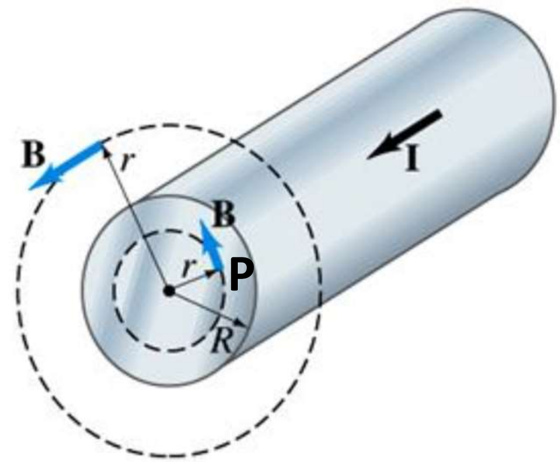
According to Biot – Savart's law, the magnetic induction at point P due to a current carrying conductor is



$$B = \frac{\mu_0 i}{2\pi d}$$

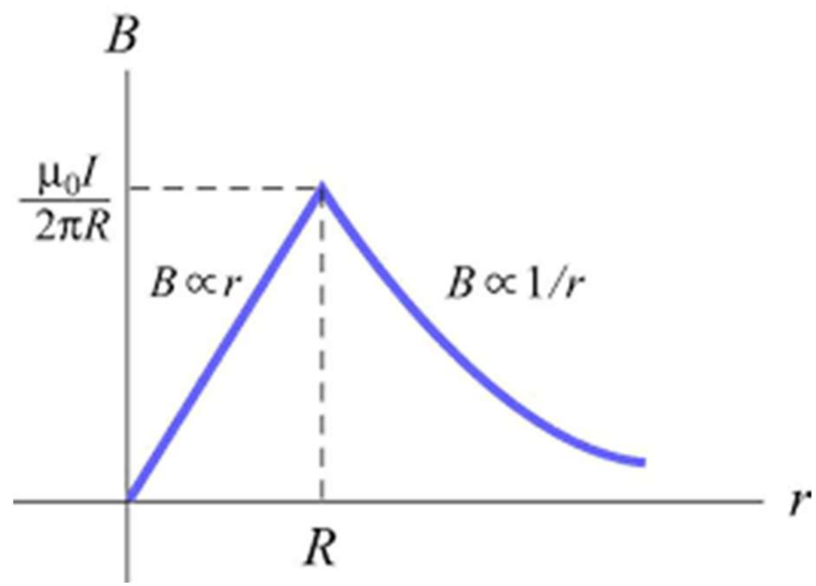
$\therefore$ *magnetic induction*

$$at\ inside\ point\ B = \frac{\mu_0\, i_{enclosed}}{2\pi r}$$

$$i_{enclosed} = \frac{i\, r^2}{R^2} \qquad \therefore \quad B = \frac{\mu_0 \frac{i\, r^2}{R^2}}{2\pi r}$$

$$B = \frac{\mu_0 \frac{i\,r^2}{R^2}}{2\pi r}$$

$$\therefore \quad B = \frac{\mu_0 i\,r}{2\pi R^2}$$

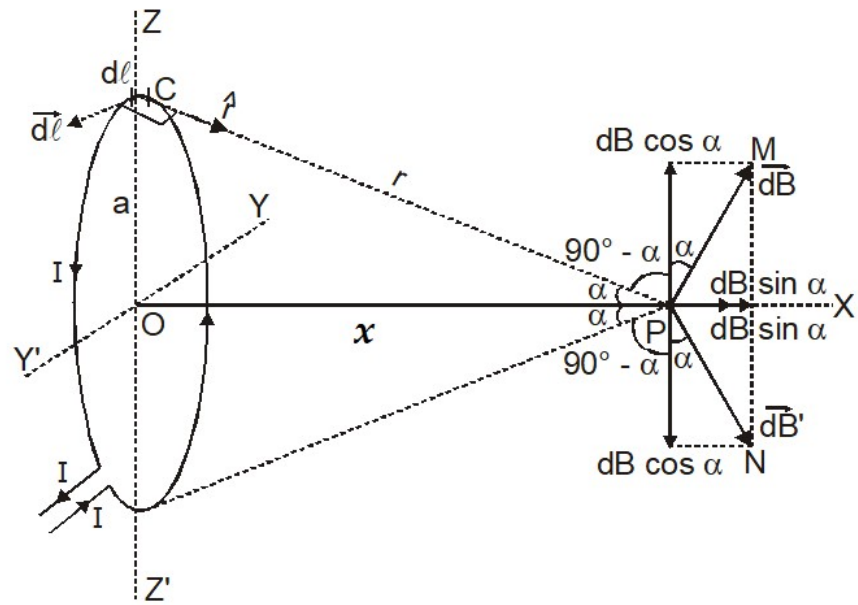


Hence, magnetic induction at a point inside a current carrying wire is directly proportional to the distance from the center of the wire to the point.

Magnetic Induction at a Point on the Axis of a Current Carrying Circular Loop

Magnetic field at an axial point of a current carrying circular loop(coil):

According to Biot-Savart's law



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$$d\bar{B} = \frac{\mu_0}{4\pi}\frac{i(d\bar{l}\,X\,\bar{r})}{r^3}$$

$$where\ \ r\ = \sqrt{a^2 + x^2}$$

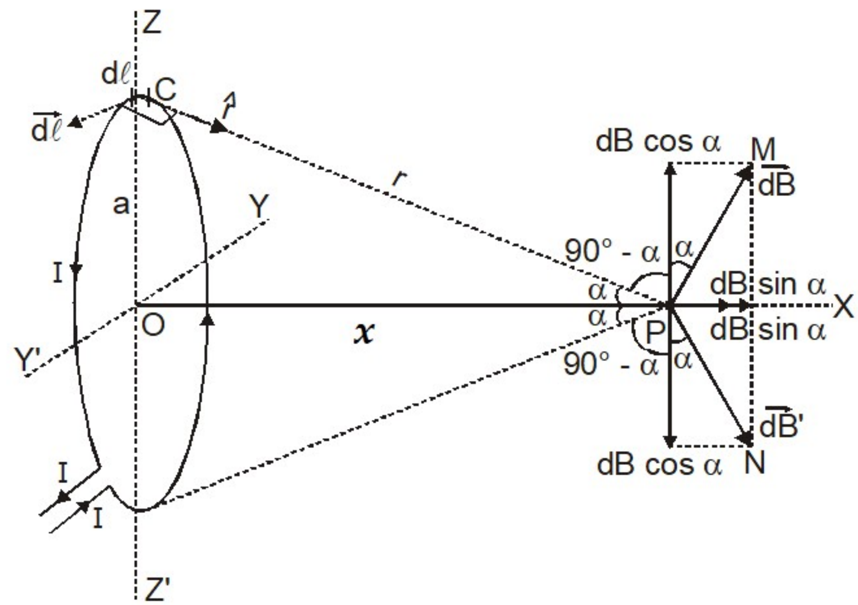
$$Here, angle\ between\ d\bar{l}\ and\ \bar{r}\ is\ 90^0$$

$$\therefore d\bar{l}\,X\,\bar{r} = dl\,r\sin 90^0 = \ dl\,r\ \ \text{(magnitude)}$$

∴Magnitude of magnetic induction

$$dB = \frac{\mu_0}{4\pi}\frac{i(d\bar{l}\ X\ \bar{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{i(dl\ r)}{r^3}$$

$$\Rightarrow \quad dB = \frac{\mu_0}{4\pi}\frac{idl}{r^2}$$

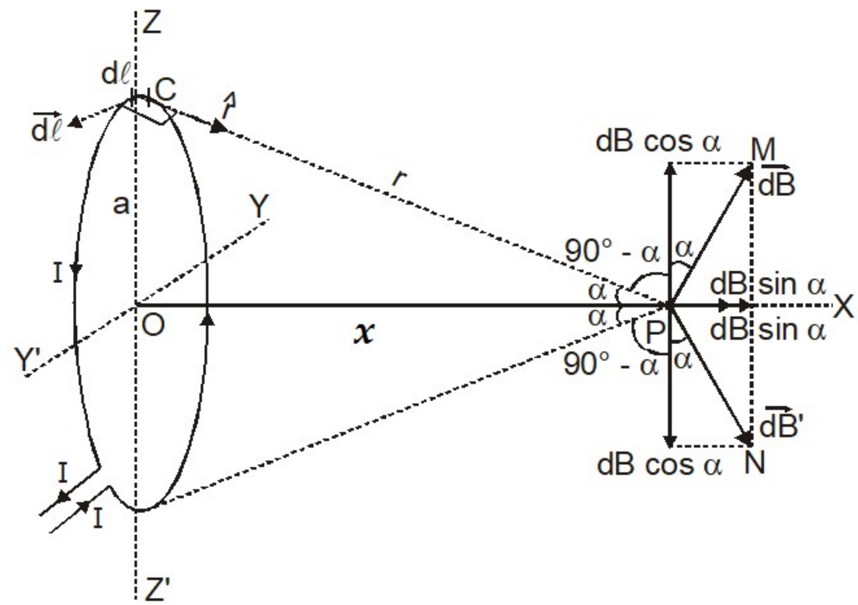


The net magnetic induction component along x – axis is

$$B = \int dB\ \sin\alpha = \int \frac{\mu_0}{4\pi}\frac{idl}{r^2}\ \sin\alpha$$

$$\Rightarrow \quad B = \frac{\mu_0 i}{4\pi}\int \frac{dl}{r^2}\sin\alpha$$

$$From\ the\ diagram\ \sin\alpha = \frac{a}{r}$$



$$\therefore B = \frac{\mu_0 i}{4\pi}\int \frac{dl}{r^2}\frac{a}{r}$$

$$\Rightarrow \quad B = \frac{\mu_0 i a}{4\pi r^3}\int dl$$

$$For\ a\ loop \int dl = 2\pi a$$

and $\quad r^3 = (a^2 + x^2)^{3/2}$

$$\Rightarrow \quad B = \frac{\mu_0 i a}{4\pi r^3} \int dl \; = \; \frac{\mu_0 i a}{4\pi\,(a^2 + x^2)^{3/2}} \; 2\pi a$$

$$\Rightarrow \quad B = \frac{\mu_0 i a^2}{2(a^2 + x^2)^{3/2}}$$

The direction of magnetic induction along the axis of the loop

If the current carrying loop is having N number of turns, then magnetic induction

$$\Rightarrow \quad B = \frac{\mu_0 N i a^2}{2(a^2 + x^2)^{3/2}}$$

Special cases:

Case 1: Magnetic induction at the center of the loop/coil

At the center, x = 0

$$\therefore B = \frac{\mu_0 N i a^2}{2(a^2+x^2)^{3/2}} = \frac{\mu_0 N i a^2}{2(a^2+0^2)^{3/2}}$$

$$\Rightarrow \quad B = \frac{\mu_0 N i a^2}{2(a^2)^{3/2}}$$

$$\Rightarrow \quad B = \frac{\mu_0 N i a^2}{2a^3} \qquad \Rightarrow \quad B = \frac{\mu_0 N i}{2a}$$

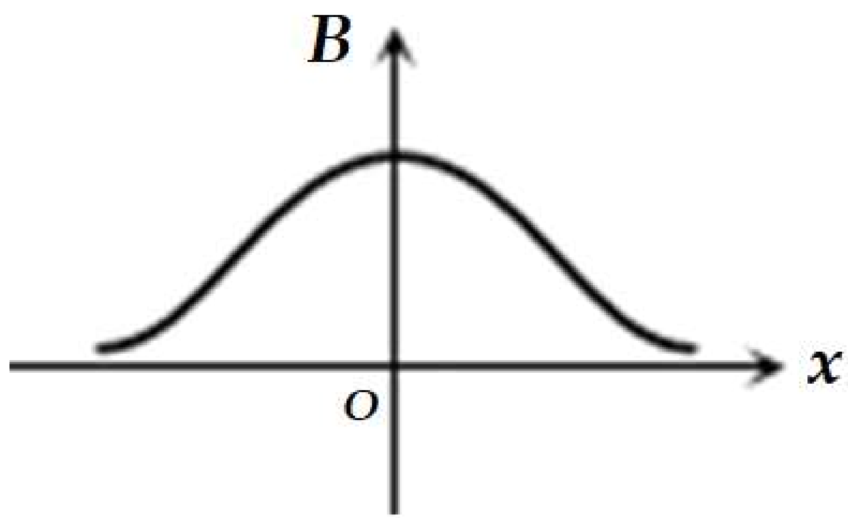
Case 2: If x >> a then

$$\therefore B = \frac{\mu_0 N i a^2}{2(a^2+x^2)^{3/2}} = \frac{\mu_0 N i a^2}{2(x^2)^{3/2}}$$

$$\Rightarrow B = \frac{\mu_0 N i \pi a^2}{2\pi x^3}$$

$$\Rightarrow B = \frac{\mu_0 N i A}{2\pi x^3} \quad where\ \ A = \pi a^2, area\ of\ the\ loop.$$

The magnetic field B varies non linearly with distance x from center of the circular loop as shown in figure



Problem(1): A 2 ampere current is flowing through a circular coil of radius 10 cm containing 100 turns. Find the magnetic flux density at the center of the coil.

Solution: Given

Current $i = 2$ *ampere*

Radius of the coil $r = 10\ cm = 10 \times 10^{-2}\ m = 10^{-1}\ m$

Number of turns in the coil $N = 100$ *turns*

Magnetic Induction $B = ?$

$Magnetic\ induction\ at\ the\ center\ of$

$$the\ coil\ B = \frac{\mu_0 N i}{2r}$$

$$B = \frac{4\pi\ X\ 10^{-7}\ X\ 100\ X\ 2}{2\ X\ 10^{-1}}$$

$$= 4\ X\ 3.14\ X\ 10^{-6} X\ 100$$

$$\therefore\ B = 12.6\ X\ 10^{-4}\ wb/m^2\ or\ tesla$$

$$= 12.6\ gauss$$

Problem (2): The magnetic induction at the center of a circular current carrying conductor of radius r is B_c. The magnetic field on its axis at a distance r from the center is B_a. Find the value of B_c/B_a.

Solution:

Given, Radius of the current carrying conductor = r

Magnetic induction at the center of the circular current carrying conductor = B_c

Distance from the center to point where magnetic induction is B_a = x = r

Magnetic induction at center of the current carrying conductor

$$B_c = \frac{\mu_0 i}{2r}$$

Magnetic induction at a point on the axis

$$B_a = \frac{\mu_0 i r^2}{2(r^2 + x^2)^{3/2}}$$

$$B_a = \frac{\mu_0 i r^2}{2(r^2 + r^2)^{3/2}}$$

$$B_a = \frac{\mu_0 i r^2}{2(r^2 + r^2)^{3/2}} = \frac{\mu_0 i r^2}{2(2r^2)^{3/2}}$$

$$\Rightarrow \ B_a = \frac{\mu_0 i r^2}{2(2)^{3/2} r^3}$$

$$\Rightarrow \ B_a = \frac{\mu_0 i}{2r(2)^{3/2}}$$

$$\therefore \quad \frac{B_c}{B_a} = \frac{\dfrac{\mu_0 i}{2r}}{\dfrac{\mu_0 i}{2r(2)^{3/2}}}$$

$$\Rightarrow \quad \frac{B_c}{B_a} = 2^{3/2}$$

THANK YOU